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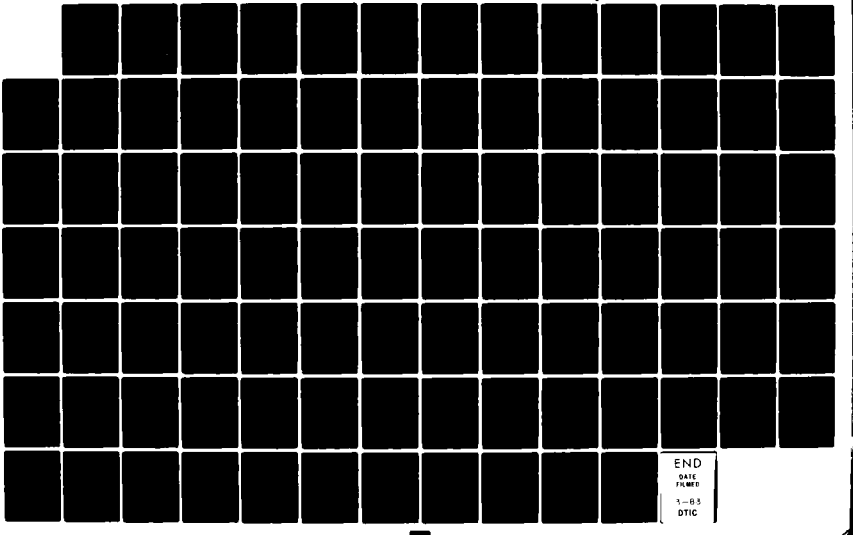

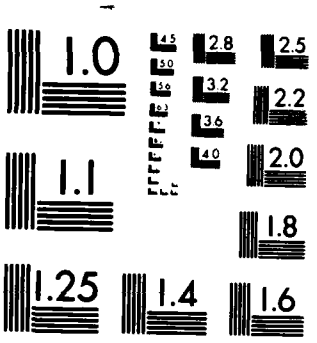

A Sensitivity Analysis of The Logarithmic-Poisson-Gamma Distribution

by

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The Logarithmic-Poisson-Gamma (LPG) distribution describes the probability distribution for demand in a lead time for the situation in which customers arrive according to a Poisson process, requisition sizes are described by the Logarithmic distribution, and lead times are Gamma distributed. This paper presents plots of the probability mass functions for the LPG and for an associated scaled binomial approximation to the LPG.

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Section I

Introduction

In Reference 1, we develop the probability distribution for demand in a lead time for the situation in which customers arrive according to a Poisson process and each customer requires X units, where X is described by the logarithmic distribution; i.e. the probability mass function of X is

$$f(x) = \frac{\Theta^x}{x \ln \Theta} \qquad x = 1, 2, \ldots$$

In addition, lead times are assumed to be Gamma Distributed. We call the resulting distribution of demand in a lead time the Logarithmic-Poisson-Gamma (LPG) distribution. This distribution appears to be a useful approximation to the demand processes of a number of Air Force EOQ-type (consumable) items.

Analytic details of the LPG distribution are presented in Reference 1. In addition, reference 1 presents a scaled negative binomial approximation to the LPG. We assume the reader is familiar with these results. In this paper, we present a sensitivity analysis of the LPG distribution and of the associated scaled negative binomial approximation.

SECTION II

Sensitivity Analysis

To get a feel for the shape of the LPG distribution, and to evaluate the accuracy of the scaled negative Binomial approximation, we computed the cumulative distribution function values for both of these distributions for 40 different parameter sets. The BASIC computer code used for these calculations is presented in Appendix B, while Figure II-1 illustrates the results of our calculations for one of these data sets. This figure presents the cumulative distribution function (CDF) and probability density function (PDF) for both the LPG and scaled negative binomial distributions for the case in which the average requisition size is 2 units, the average monthly demand rate is .5, the average lead time is 8 months, and the coefficient of variation of lead time is .25. For this situation, the mean, coefficient of variation, skewness, and kurtosis of the distribution of demand in a leadtime equals 4.00, 0.97, 1.46, and 6.40, respectively. These values are shown on the left hand side of Figure II-1. The specific parameters utilized in the LPG distribution calculations are also shown on the left hand side of this figure, as well as specific CDF values. The column labeled "FX" presents cumulative distribution function values for the LPG distribution, while the column labeled "FNBX" presents the corresponding cumulative distribution function values for the scaled negative binomial approximation. Finally, the column labeled "DIFF" presents the difference between the LPG distribution values and the corresponding negative binomial CDF values.

The right hand side of Figure II-1 plots both the cumulative and density function values associated with the LPG and scaled negative binomial distributions. As shown in

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

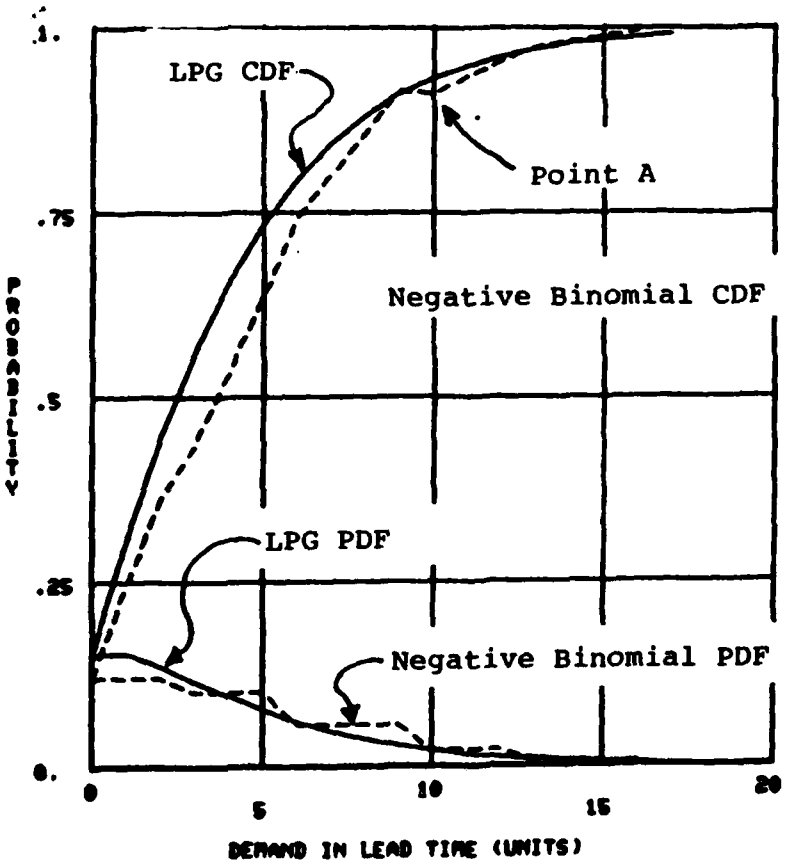
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FMAX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.78	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



Scaled Negative Binomial CDF

LPG CDF

-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

Figure II-1. LPG and Scaled Negative Binomial Distributions for the LO Demand Base Case.

the Figure, the scaled negative binomial approximation underestimates the corresponding LPG CDF values for low levels of demand in the leadtime, but the approximation becomes much better for demand values exceeding 10 units. Note the slight down-turn labeled as Point A. This is caused by the rounding rules employed in the scaled negative binomial code. A slight change in these rules may be used to make the CDF monotonic.

Figure II-2 presents a similar set of calculations in which the monthly demand rate is equal to 2 units per month, and the other parameters are the same as used in Figure II-1. In this case, the scaled negative binomial distribution is a much closer approximation to the LPG curve.

We will refer to the specific data sets plotted in Figures II-1 and II-2 as the "LO" and "HI" base cases, respectively, Table II-1 presents the specific parameter values selected for these calculations. The data set numbers assigned to each of these curves are shown in the right hand side of Table II-1 and detailed plots associated with each of the data sets are presented in Appendix A. Let us now discuss our results.

Results for LO Base Case.

As noted above, the LO Base Case has the following characteristics:

	Value
Average Requisition Size (Units)	2
Average Units Demanded per Month	.5
Average Lead Time (Months)	8
Coefficient of Variation of Lead Time	.25

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

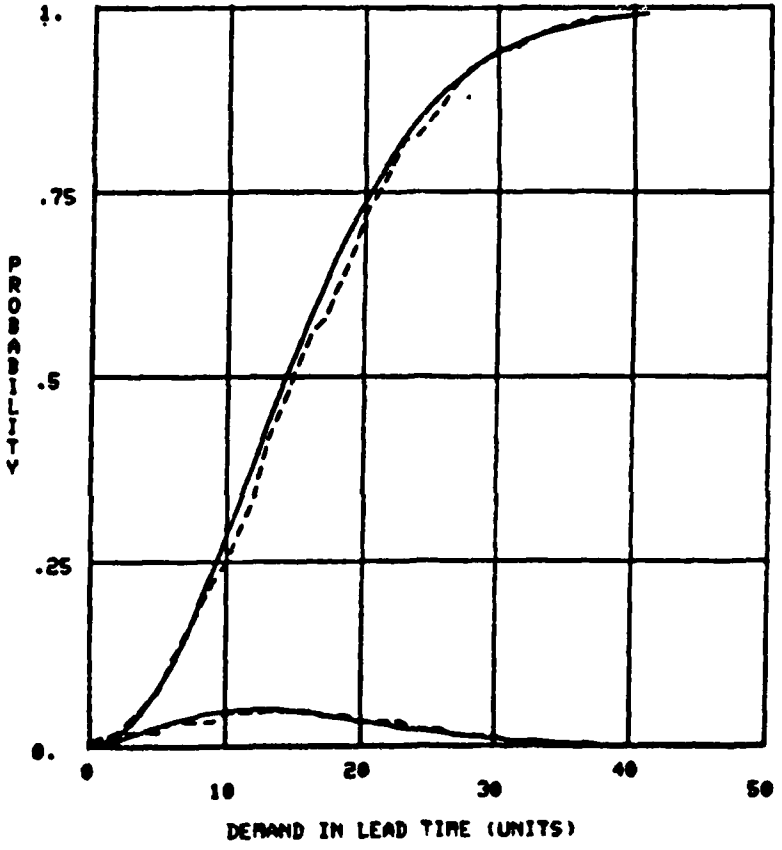
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

Figure II-2. LPG and Scaled Negative Binomial Distributions for the HI Demand Base Case.

TABLE II-1

Sensitivity Analysis Parameter Values

Variable	Parameter Settings	Data Set Number	
		LO	HI
Average Requisition size	1.01, 2, 4, 8, 16	1-5	21-25
Average Lead Time in Months	4, 6, 8, 16	6-10	26-30
Demand Rate in Units per Month	.1, .5, 1., 1.5, 2.0	11-15	31-35
Coefficient of Variation of Leadtime	.01, .25, .50, .75, 1.0	16-20	36-40

Let us now consider the impact of variations in these characteristics.

Requisition Size Sensitivity. In performing our sensitivity analyses, three of the major distribution characteristics defined in Table II-1 were held constant, while the fourth characteristic was varied. For example, data set numbers 1 thru 5 were constructed by holding average leadtime, demand rate, and the coefficient of variation of leadtime equal to the values for the LO demand base case, while varying the average requisition size. Data set numbers 21 thru 25 were performed similarly, but in this case the HI demand base case was used.

Details of our results for all 40 data sets are presented in Appendix A, while Table II-2 and Figures II-3 thru II-7 illustrate our results. Table II-2 summarizes the primary calculations associated with data sets 1 thru 5, while Figures II-3 thru II-7 present the associated plots of the LPG CDF. The table summarizes the sensitivity of the primary LPG distribution parameters (i.e. Theta Lambda, Alpha, and Beta) to changes in average requisition size for the low demand case base. This table displays the requisition size, demand rate, mean leadtime and leadtime variability coefficients that were used to drive the data set calculations, as well as the associated LPG parameters and the moments of the LPG leadtime demand distribution. Finally, at the bottom of Table II-2, we present the specific demand values associated with given percentage points of the LPG distribution and of the associated scaled negative binomial approximation. For example, consider the data shown in the first column at the bottom of Table II-2. For data set number 1, a reorder point of eight units is required to insure a 95% probability of no stockouts during a leadtime. Using the scaled negative binomial approximation for data set 1, a reorder level of 9 units is required to produce the same 95%

SAMPLE LPG CALCULATIONS

Data Set No.	1	2	3	4	5
Ave. Req. Size	1.01	2	4	8	16
Ave. Demand/Mo.	.5	.5	.5	.5	.5
Mean Lead Time	8.0	8.0	8.0	8	8
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.02	.72	.90	.96	.99
Lambda	.49	.25	.12	.06	.03
Alpha	16	16	16	16	16
Beta	2	2	2	2	2
LPG Moments					
Mean	4	4	4	4	4
CV	.56	.97	1.63	2.63	4.15
Skewness	.41	1.46	2.96	5.08	8.18
Kurtosis	3.23	6.40	16.45	42.13	103.79

	1		2		3		4		5	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	4	5	3	4	1	4	0	8	0	18
.60	4	5	4	5	2	5	0	10	0	22
.70	5	6	5	6	4	6	2	11	0	26
.80	6	7	7	7	7	10	4	13	1	29
.85	6	7	8	8	9	12	7	14	3	31
.90	7	8	9	9	12	14	12	22	7	33
.95	8	9	12	12	17	16	22	33	22	43
.97	9	10	13	13	21	22	31	38	38	68
.99	10	11	17	16	31	26	52	42	82	93

Table II-2. Requisition Size Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	1.01
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

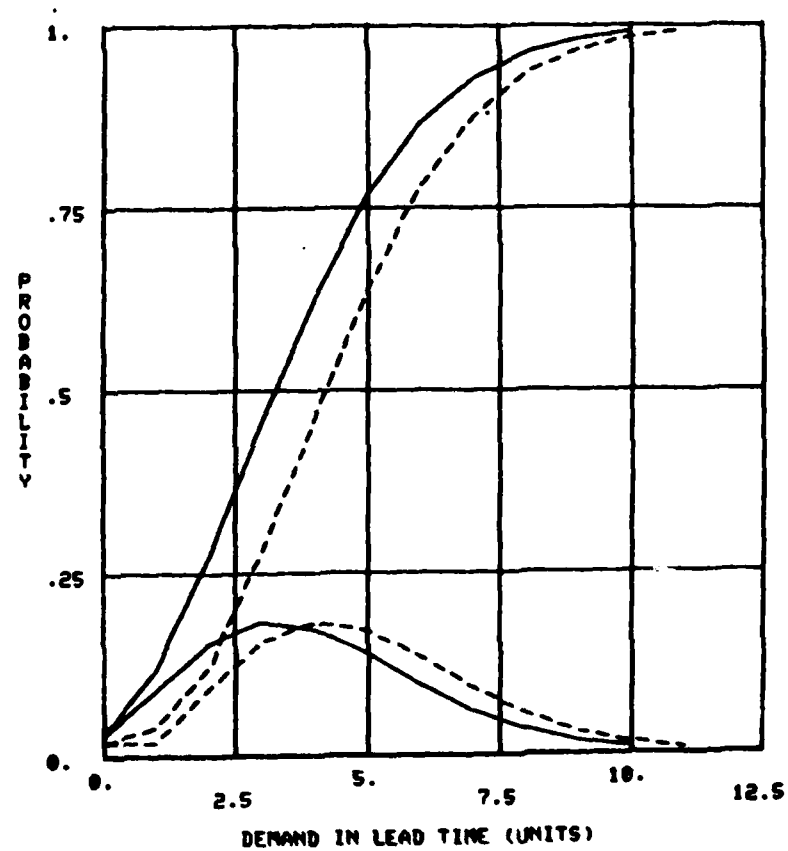
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.56
SKEWNESS	0.41
KURTOSIS	3.23

LPG PARAMETERS

THETA	0.02
LAMBDA	0.49
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.03	0.02	0.01
1.	0.12	0.04	0.08
2.	0.27	0.12	0.15
3.	0.46	0.28	0.18
4.	0.63	0.47	0.16
5.	0.77	0.64	0.13
6.	0.87	0.78	0.09
7.	0.93	0.87	0.05
8.	0.96	0.93	0.03
9.	0.98	0.97	0.02
10.	0.99	0.98	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.177 FOR X = 3

Figure II-3. Data Set No. 1 LPG Curves.

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

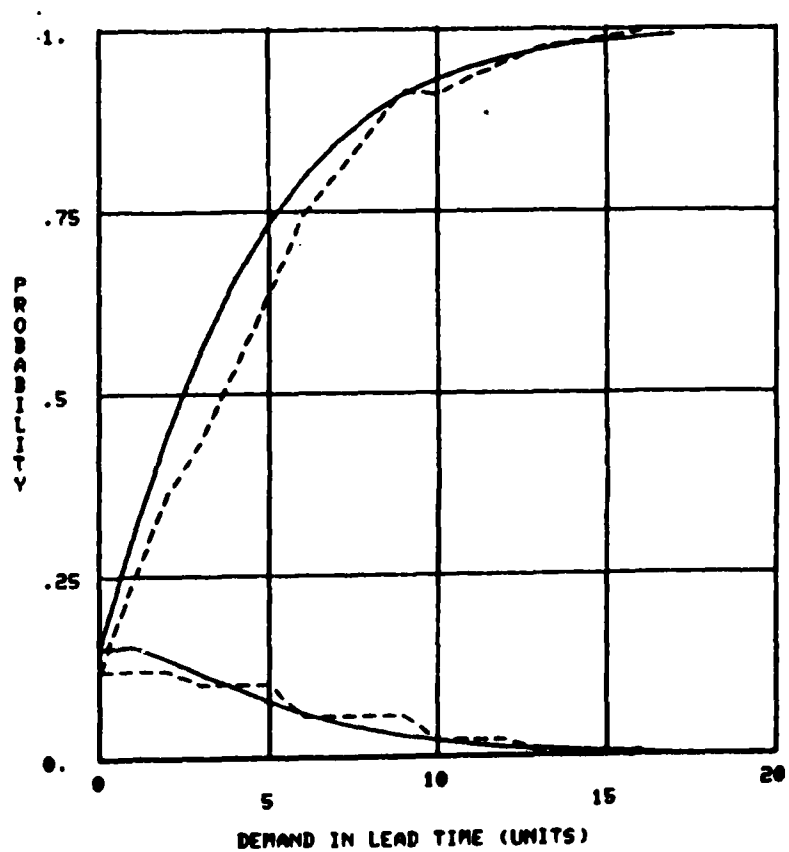
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

Figure II-4. Data Set No. 2 LPG Curves.

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	4.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	1.63
SKEWNESS	2.96
KURTOSIS	16.45

LPG PARAMETERS

THETA	0.90
LAMBDA	0.12
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.38	0.11	0.27
1.	0.52	0.22	0.30
2.	0.61	0.33	0.27
3.	0.67	0.44	0.23
4.	0.72	0.55	0.17
5.	0.76	0.66	0.10
6.	0.80	0.71	0.09
7.	0.82	0.73	0.09
8.	0.85	0.76	0.09
9.	0.87	0.78	0.08
10.	0.88	0.81	0.08
11.	0.90	0.83	0.07
12.	0.91	0.86	0.05
13.	0.92	0.88	0.04
14.	0.93	0.91	0.02
15.	0.94	0.93	0.01
16.	0.95	0.96	-0.01
17.	0.95	0.95	0.01
18.	0.96	0.95	0.01
19.	0.96	0.96	0.01
20.	0.97	0.96	0.01

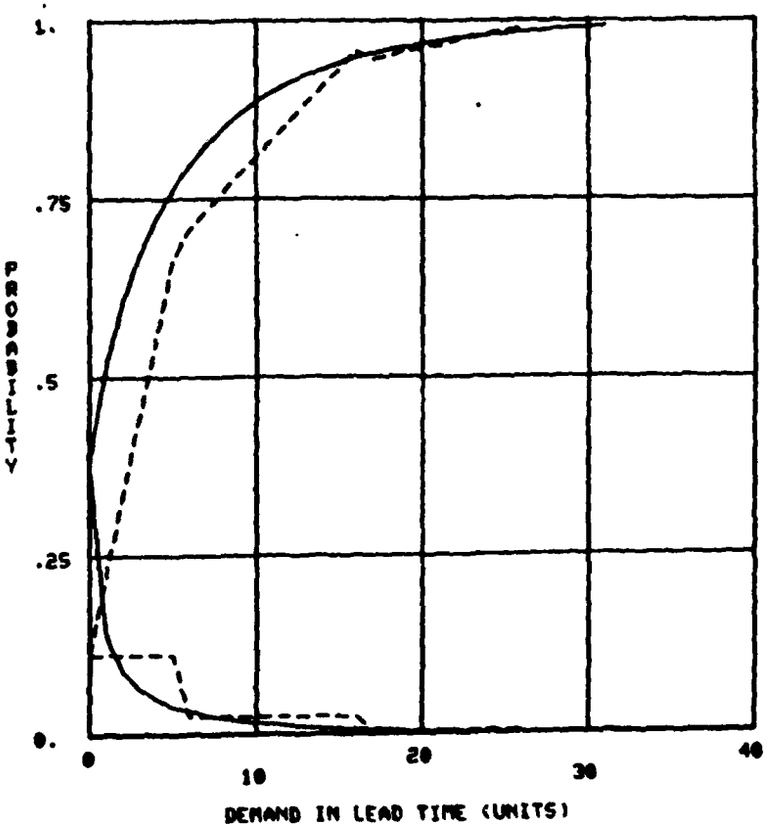


Figure II-5. Data Set No. 3 LPG Curves.

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	8.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	2.63
SKEWNESS	5.08
KURTOSIS	42.13

LPG PARAMETERS

THETA	0.96
LAMBDA	0.06
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.61	0.06	0.55
1.	0.70	0.12	0.58
2.	0.75	0.18	0.57
3.	0.78	0.23	0.54
4.	0.80	0.29	0.51
5.	0.82	0.35	0.47
6.	0.84	0.41	0.43
7.	0.86	0.47	0.39
8.	0.87	0.53	0.34
9.	0.88	0.59	0.29
10.	0.89	0.64	0.24
11.	0.90	0.70	0.19
12.	0.90	0.76	0.14
13.	0.91	0.82	0.09
14.	0.92	0.88	0.04
15.	0.92	0.87	0.05
16.	0.93	0.87	0.05
17.	0.93	0.88	0.05
18.	0.94	0.88	0.05
19.	0.94	0.89	0.05
20.	0.94	0.89	0.05

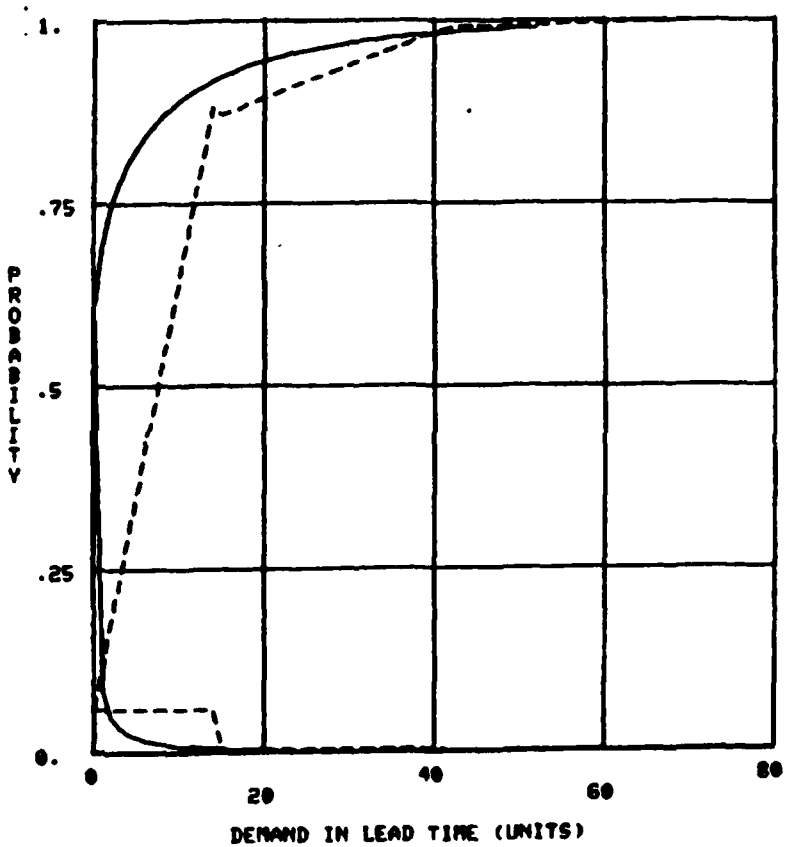


Figure II-6. Data Set No. 4 LPG Curves.

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	16.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

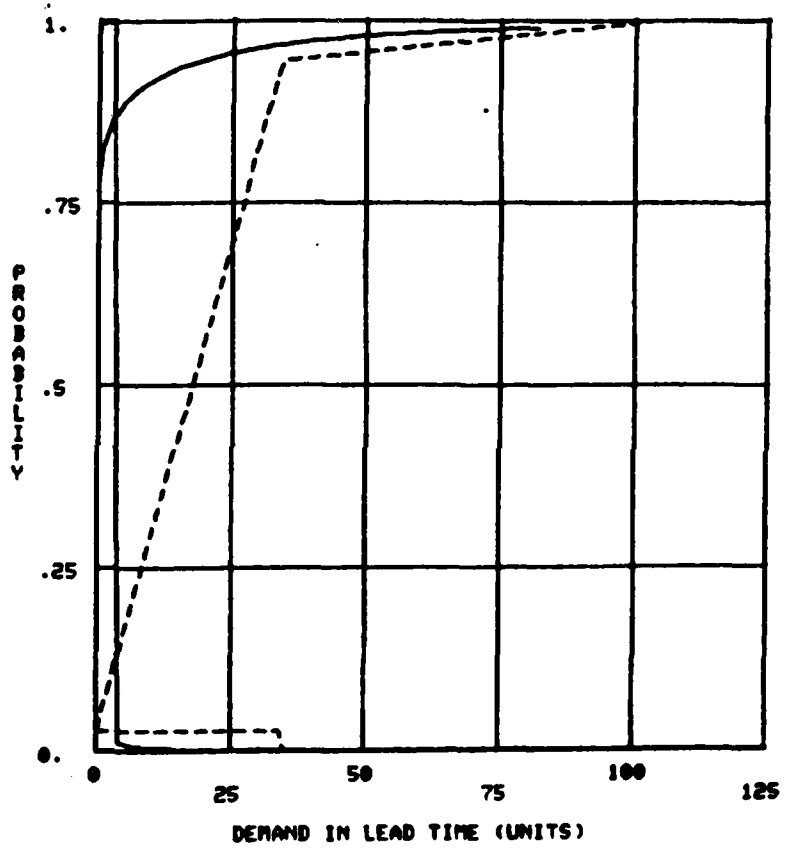
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	4.15
SKEWNESS	8.18
KURTOSIS	103.79

LPG PARAMETERS

THETA	0.99
LAMBDA	0.03
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.78	0.03	0.75
1.	0.83	0.05	0.77
2.	0.85	0.08	0.77
3.	0.86	0.11	0.76
4.	0.88	0.13	0.74
5.	0.89	0.16	0.73
6.	0.89	0.19	0.71
7.	0.90	0.21	0.69
8.	0.91	0.24	0.67
9.	0.91	0.27	0.64
10.	0.92	0.29	0.62
11.	0.92	0.32	0.60
12.	0.92	0.35	0.58
13.	0.93	0.37	0.55
14.	0.93	0.40	0.53
15.	0.93	0.43	0.51
16.	0.94	0.45	0.48
17.	0.94	0.48	0.46
18.	0.94	0.51	0.43
19.	0.94	0.53	0.41
20.	0.95	0.56	0.39



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.772 FOR X = 1

Figure II-7. Data Set No. 5 LPG Curves.

probability. Scanning the results for all five data sets, we observe that the scaled negative binomial provides reorder point results very similar to the LPG distribution for average requisition sizes of 4 or less, but provides a very poor approximation for the high average requisition size values.

Lead Time Sensitivity. Table II-3 presents percentage points for the LPG and Scaled Negative Binomial (SNB) approximations for the LO Base case and Data Sets 6 thru 10. This Table shows the effect of mean lead times of 4, 6, 8, 10, and 12 months, respectively. Note that the SBN yields percentage points which are very close to the LPG values, and that the approximation improves as the lead time increases.

Demand Rate Sensitivity. Table II-4 presents results of varying the LO Base case using demand rates of .1, .5, 1.0, 1.5, and 2.0 units per month, respectively, Note that the LPG and SNB values are vary similar for all but the demand =.1 case.

Lead Time Variability Sensitivity. Table II-5 presents the results of increasing lead time variability (Dat Sets 16-20). Observe the percentage points are within 1 unit for all but the CV=.75, 60% point. Hence, the approximation is excellent for all five data sets.

Results for the HI Base Case. The HI Base Case has the following characteristics:

	Value
Average Requisition Size	2
Average Units Demanded per Month	2
Average Lead Time (Months)	8
Coeeficient of Variation of Leat Time	.25

SAMPLE LPG CALCULATIONS

Data Set No.	6	7	8	9	10
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.50	.50	.50	.50	.50
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33
LPG Moments					
Mean	2.00	3.00	4.00	5.00	6.00
Skewness	2.16	1.72	1.46	1.28	1.14
Kurtosis	10.22	7.67	6.40	5.65	5.15

Percentage Points	6		7		8		9		10	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	1	2	2	3	3	4	4	5	5	6
.60	2	2	3	4	4	5	5	6	6	7
.70	2	4	4	5	5	6	6	7	8	8
.80	3	5	5	6	7	7	8	9	9	11
.85	4	5	6	7	8	8	9	9	11	12
.90	5	6	7	8	9	9	11	12	12	13
.95	7	8	10	9	12	12	14	14	15	15
.97	9	9	11	12	13	13	15	15	17	16
.99	12	9	15	14	17	16	19	19	22	20

Table II-3. Lead Time Sensitivity for LO Base Case

SAMPLE LPG CALCULATIONS

Data Set No.	11	12	13	14	15
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.93	3.64

	11		12		13		14		15	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	26	26
.90	3	5	9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	13	21	21	28	28	35	35
.99	8	8	17	16	26	25	34	33	41	40

Table II-4. Demand Rate Sensitivity for LO Base Case

SAMPLE LPG CALCULATION

Data Set No.	16	17	18	19	20
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	0.5	0.5	0.5	0.5	0.5
Mean Lead Time	8	8.0	8.0	8.0	8.0
CV of Lead Time	.01	.25	.50	.75	1.0
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	1,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13
LPG Moments					
Mean	4.00	4.00	4.00	4.00	4.00
CV	.94	.97	1.06	1.20	1.37
Skewness	1.61	1.46	1.21	1.13	1.29
Kurtosis	6.84	6.40	5.55	5.15	5.76

	16		17		18		19		20	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	3	4	3	4	3	4	2	3	2	2
.60	4	5	4	5	4	5	3	5	3	4
.70	5	6	5	6	5	6	5	6	5	6
.80	7	7	7	7	7	8	7	8	7	8
.85	8	8	8	8	8	9	8	9	9	9
.90	9	9	9	9	10	11	10	11	11	12
.95	11	12	12	12	12	13	14	14	15	15
.97	13	13	13	13	14	14	16	17	18	19
.99	17	15	17	16	19	19	21	21	25	25

Table II-5. Lead Time Variability Sensitivity for LO Base Case

Recall that Figure II-2 plots the LPG and SNB curves for this case, and that the detailed plots for each data set are presented in Appendix A.

Requisition Size Sensitivity. Table II-6 presents the results for Data Sets, 21-25, reflecting average requisition sizes of 1.01, 2.0, 4.0, 8.0, and 16.0, respectively. Note that the LPG and SNB curves yield similar results for average requisition sizes of 4 or less, but differ significantly for average requisition sizes of 8 and 16.

Lead Time Sensitivity. Table II-7 presents results for Data Sets 26-30, reflecting lead times of 4, 6, 8, 10, and 12 months, respectively. For this case, the LPG and SNB yield similar results in all cases.

Demand Rate Sensitivity. The results for Data Sets 31-35 are presented in Table II-8. These Data Sets have average demands of .1, .5, 1, 1.5, and 2 units per month, respectively. Observe that the SNB approximation is a particularly good fit to the LPG except for the 50 and 60% points for the .1 units per month case.

Lead Time Variability Sensitivity. Table II-9 presents results for coefficients of variation of .01, .25, .50, .75, and 1.0, respectively (Data Sets 36-40). Note that the LPG and SNB distributions are very similar for all of these cases.

SAMPLE LPG CALCULATIONS

Data Set No.	21	22	23	24	25
Ave. Req. Size	1.01	2.00	4.00	8.00	16.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.02	.72	.90	.96	.99
Lambda	1.98	1.00	.50	.25	.13
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	16.00	16.00	16.00	16.00	16.00
CV	.36	.53	.84	1.33	2.09
Skewness	.27	.60	1.35	2.44	4.02
Kurtosis	3.13	3.64	5.97	12.32	27.79

Percentage Points	21		22		23		24		25	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	15	16	15	15	13	14	8	13	2	22
.60	17	18	17	18	16	19	12	18	5	26
.70	19	19	19	20	20	23	18	26	11	31
.80	21	21	23	23	25	27	27	35	23	37
.85	22	22	25	26	29	31	33	39	33	56
.90	24	24	27	28	34	35	43	47	50	75
.95	26	27	32	32	42	42	59	61	80	93
.97	28	28	35	34	49	46	72	67	105	101
.99	31	32	41	40	61	57	99	88	163	148

Table II-6. Requisition Size Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

Data Set No.	26	27	28	29	30
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33
LPG Moments					
Mean	8.00	12.00	16.00	20.00	24.00
CV	.71	.60	.53	.49	.46
Skewness	.95	.73	.60	.52	.47
Kurtosis	4.53	3.92	3.64	3.48	3.38

	26		27		28		29		30	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	7	8	11	12	15	15	19	20	23	23
.60	8	9	13	13	17	18	21	22	25	26
.70	10	11	15	15	19	20	24	25	29	29
.80	12	13	17	19	23	23	28	28	33	33
.85	14	14	19	20	25	26	30	30	35	36
.90	16	16	22	22	27	28	33	34	39	39
.95	19	19	25	26	32	32	38	37	44	44
.97	21	21	28	28	35	34	41	41	48	48
.99	26	25	34	33	41	40	48	48	55	55

Table II-7. Lead Time Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

Data Set No.	31	32	33	34	35
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosos	21.73	6.40	4.53	3.92	3.64

	31		32		33		34		35	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	25	26
.90	3	5	1 9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	3	21	21	28	28	35	34
.99	8	8	17	16	26	25	34	33	41	40

Table II-8. Demand Rate Sensitivity for HI Base Case

SAMPLE LPG CALCULATIONS

Date Set No.	36	37	38	39	40
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.01	.25	.50	.75	1.00
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	10,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13
LPG Moments					
Mean	16.00	16.00	16.00	16.00	16.00
CV	.47	.53	.69	.88	1.10
Skewness	.80	.60	.65	1.03	1.55
Kurtosis	3.96	3.64	3.69	4.86	7.10

	36		37		38		39		40	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	15	16	15	15	14	14	12	13	10	11
.60	17	18	17	18	17	18	16	16	14	15
.70	19	20	19	20	20	20	19	20		
.80	22	22	23	23	24	25	26	23	27	27
.85	24	25	25	26	27	27	30	26	32	33
.90	26	27	27	28	31	31	35	35	39	40
.95	30	29	32	32	37	37	44	44	51	51
.97	32	32	35	34	41	41	50	50	60	61
.99	38	36	41	40	50	51	64	63	80	80

Table II-9. Lead Time Variability Sensitivity for HI Base Case

Moments of the LPG Distribution

As noted above, we computed the first four moments of the LPG distribution. Figures II-8 and II-9 present the means and standard deviations associated with each of the forty data sets studied, while Figure II-10 presents a plot of (skewness)^2 value versus the kurtosis for the forty data set calculations. Figure II-10 is particularly interesting. As shown in Johnson and Kotz (1969, Pg 39), the skewness^2-kurtosis relationships of many well known probability distributions are described by specific lines, points, or regions when plotted on a chart similar to Figure II-10. For example, the lines and regions associated with Normal, Gamma, Log-normal, Poisson, Binomial, and Negative Binomial Distributions are shown in Figure II-10. Hence, the plot of skewness^2 versus kurtosis for a specific probability distribution may be used to gain insights into appropriate approximations for that distribution. As shown in Figures II-10 thru II-13, all of the sample skewness^2-kurtosis points associated with our LPG calculations fall on a line which is almost identical with that associated with the gamma probability distribution. Hence, it appears that the gamma may be a very useful approximation for LPG calculations.

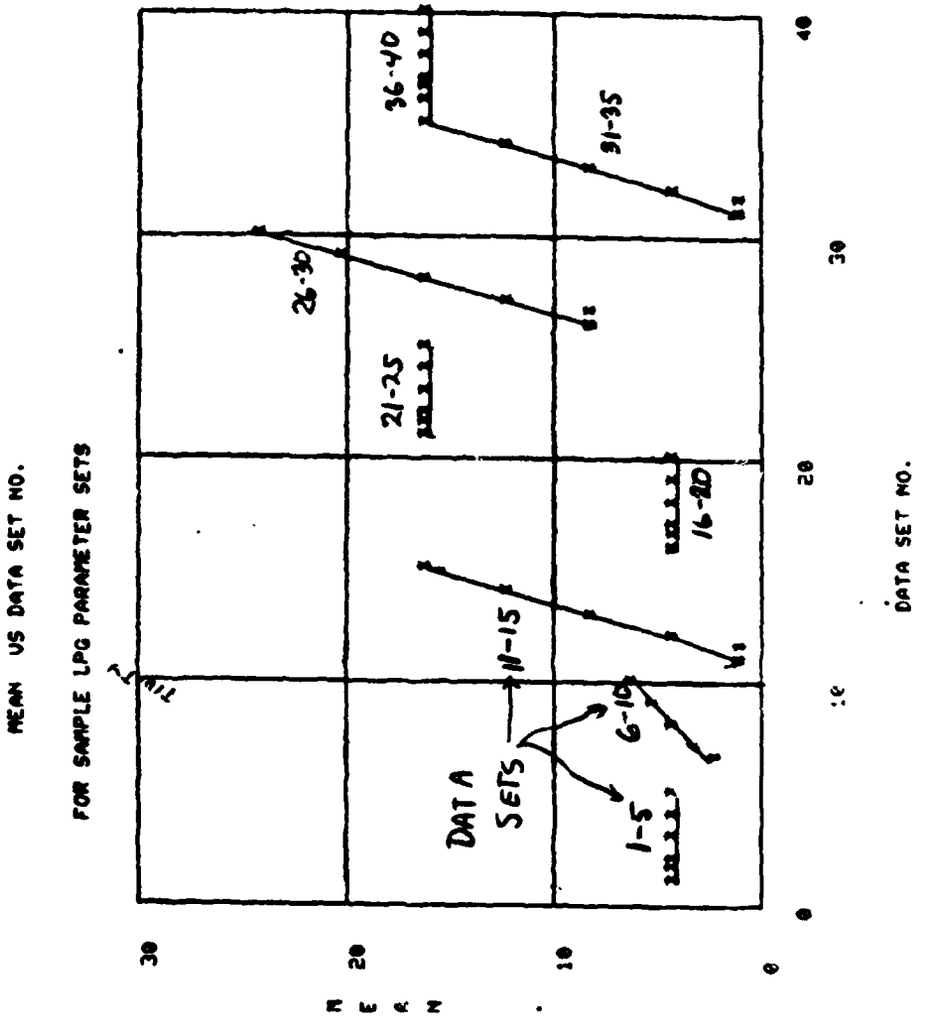


Figure II-8. Sample LPG Means.

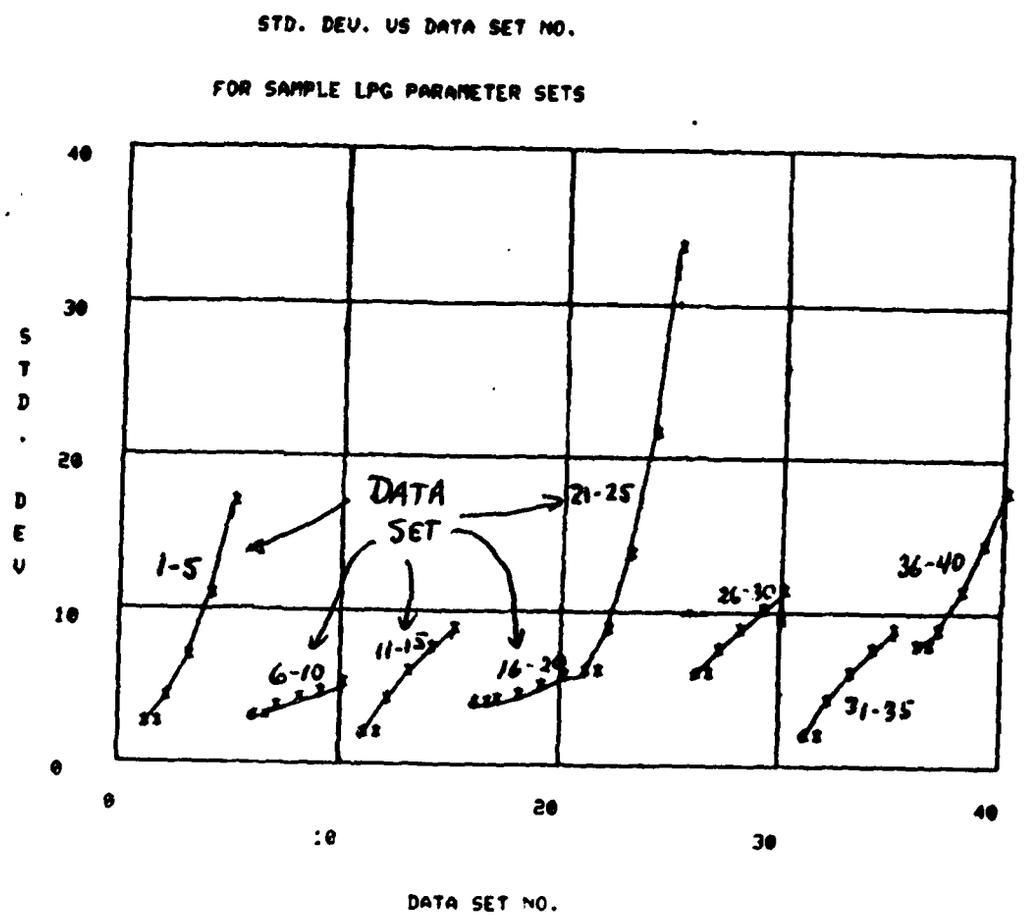


Figure II-9. Sample LPG Standard Deviation.

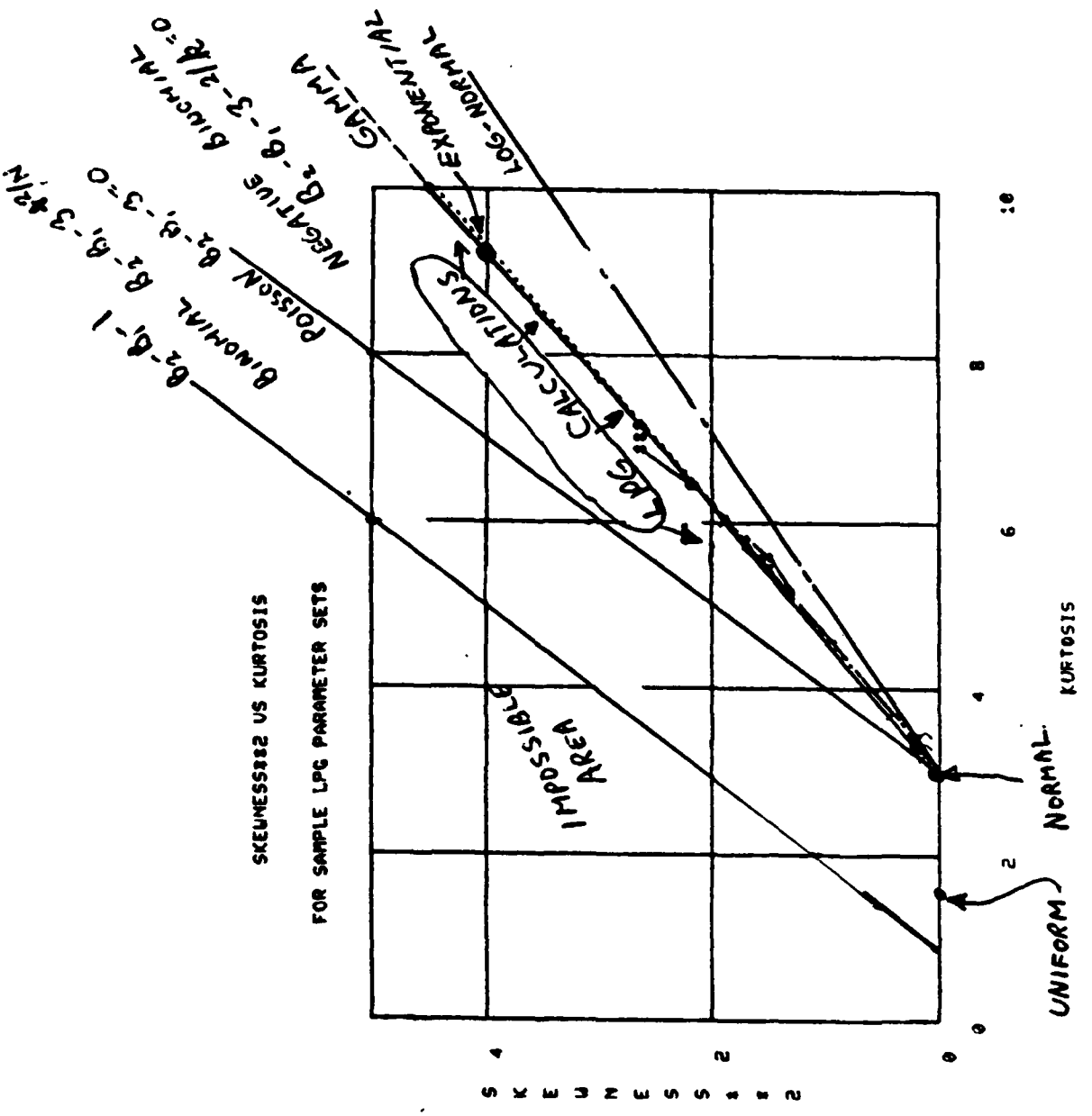


Figure II-10. Skewness^2 vs Kurtosis for Sample LPG Distribution.

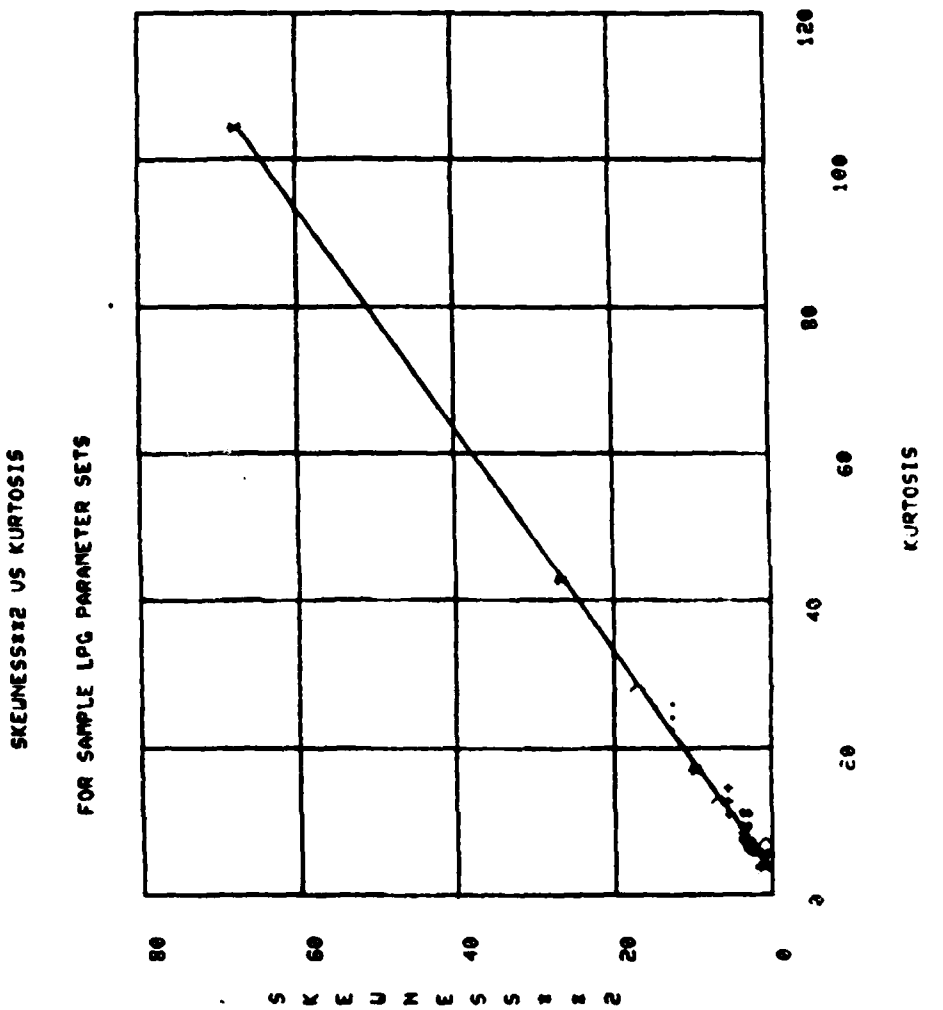


Figure II-11. Skewness^2 vs Kurtosis for Sample LPG Distribution with Unrestricted Scales.

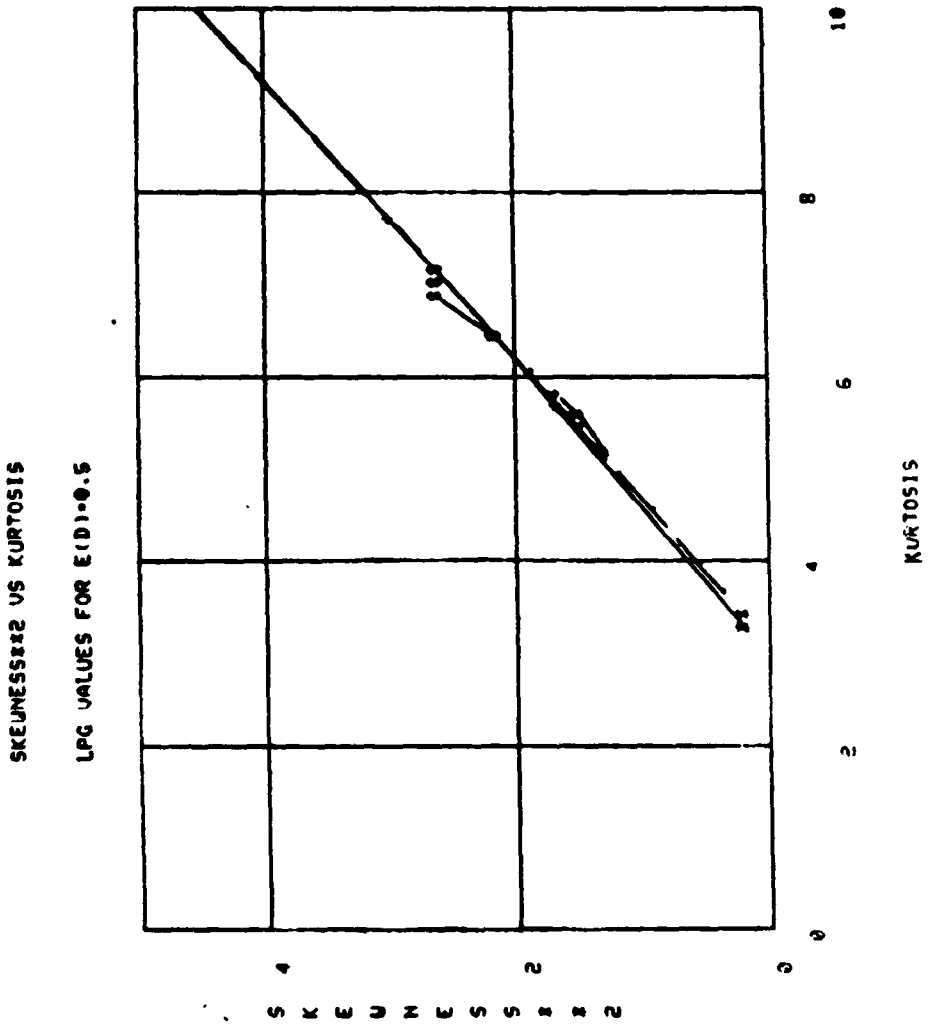


Figure II-12. Skewness^2 vs Kurtosis for LO Base Case.

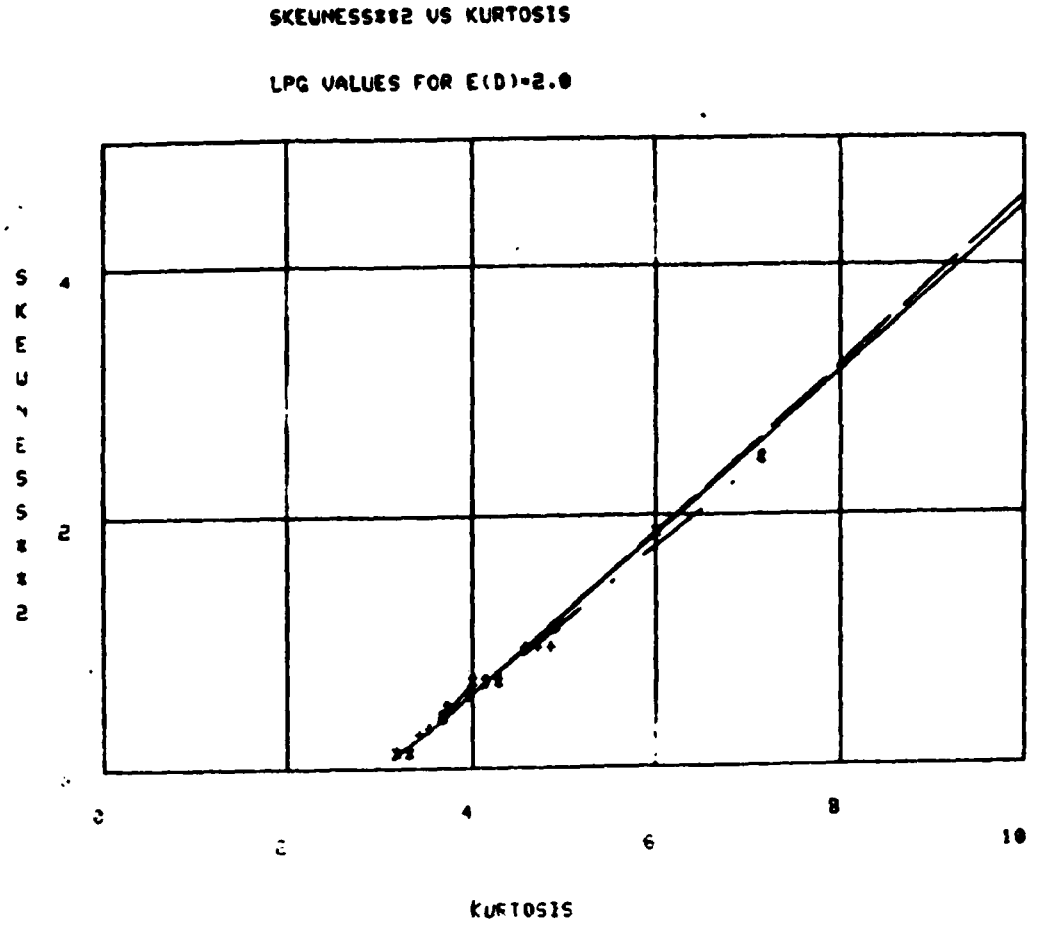


Figure II-13. Skewness^2 vs Kurtosis for HI Base Case.

Section III

Summary and Conclusions

In this paper, we have presented sample distribution functions for the Logarithmic-Poisson-Gamma (LPG) distributed and for the Scaled Negative Binomial (SNB) approximation to the LPG. Two base cases were considered; these cases have the following characteristics.

	LO Base Case	HI Base Case
Average Requisition Size (units)	2	2
Average Units Demanded per Month	.5	2
Average Lead Time (Months)	8	8
Coefficient of Variation of Lead Time	.25	.25

The effects of varying each of these parameters while holding the other three constant were investigated for base cases. We found that the SNB approximation provides percentage points very similar to the LPG except when average requisition sizes exceed 4 units or when demand per month is very small. Hence, use of the SNB approximation appears to be a promising way to significantly reduce the computer requirements associated with the LPG distribution.

We also analyzed the skewness^2 versus kurtosis relationships for the sample data sets. These relationships indicate that the continuous Gamma distribution may be an excellent approximation to the Logarithmic-Poisson-Gamma.

Section IV

References

1. Nahmias, Steven and W. Steven Demmy, The Logarithmic-Poisson-Gamma Distribution: A Model for the Distribution of Demand in a Lead Time, Working Paper WP-81-04, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Oh 45431, August 1981, 22 pp.

2. Johnson, N.L. and S. Kotz, Discrete Distributions, Wiley, 1969.

Appendix A

Detailed Item Plots

SAMPLE LPG CALCULATIONS

Data Set No.	1	2	3	4	5
Ave. Req. Size	1.01	2	4	8	16
Ave. Demand/Mo.	.5	.5	.5	.5	.5
Mean Lead Time	8.0	8.0	8.0	8	8
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.02	.72	.90	.96	.99
Lambda	.49	.25	.12	.06	.03
Alpha	16	16	16	16	16
Beta	2	2	2	2	2
LPG Moments					
Mean	4	4	4	4	4
CV	.56	.97	1.63	2.63	4.15
Skewness	.41	1.46	2.96	5.08	8.18
Kurtosis	3.23	6.40	16.45	42.13	103.79

	1		2		3		4		5	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	4	5	3	4	1	4	0	8	0	18
.60	4	5	4	5	2	5	0	10	0	22
.70	5	6	5	6	4	6	2	11	0	26
.80	6	7	7	7	7	10	4	13	1	29
.85	6	7	8	8	9	12	7	14	3	31
.90	7	8	9	9	12	14	12	22	7	33
.95	8	9	12	12	17	16	22	33	22	43
.97	9	10	13	13	21	22	31	38	38	68
.99	10	11	17	16	31	26	52	42	82	93

Table II-2. Requisition Size Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	1.01
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

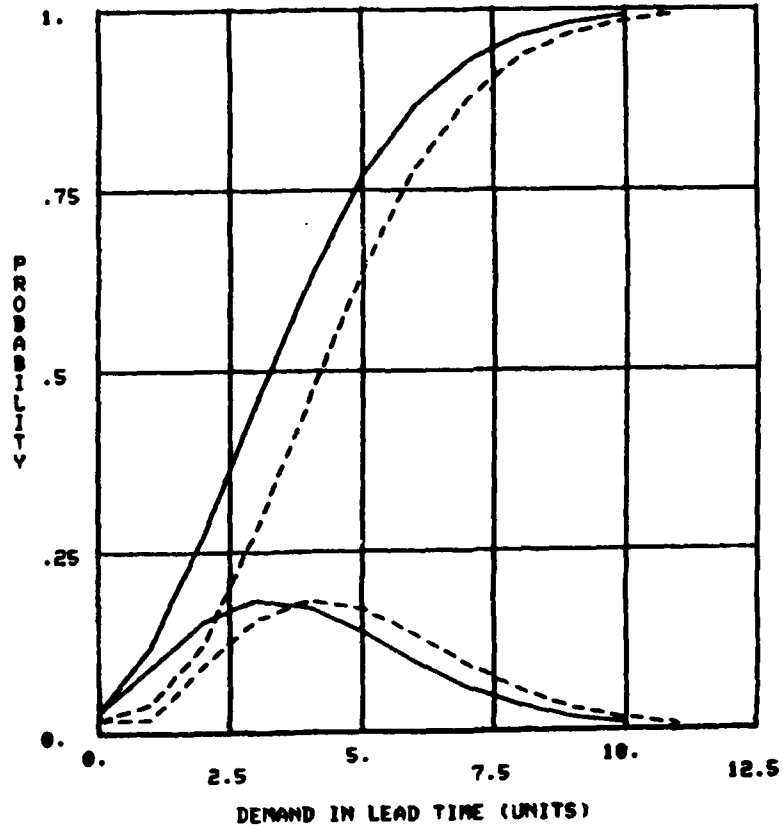
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.56
SKEWNESS	0.41
KURTOSIS	3.23

LPG PARAMETERS

THETA	0.02
LAMBDA	0.49
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.03	0.02	0.01
1.	0.12	0.04	0.08
2.	0.27	0.12	0.15
3.	0.46	0.28	0.18
4.	0.63	0.47	0.16
5.	0.77	0.64	0.13
6.	0.87	0.78	0.09
7.	0.93	0.87	0.05
8.	0.96	0.93	0.03
9.	0.98	0.97	0.02
10.	0.99	0.98	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.177 FOR X = 3

A-2-A

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

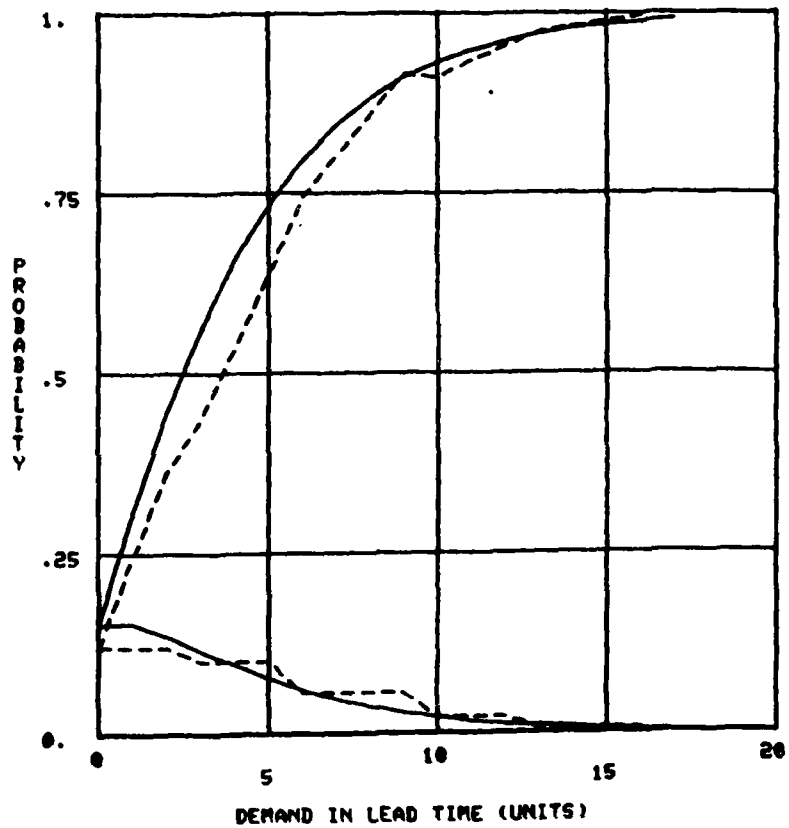
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	4.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

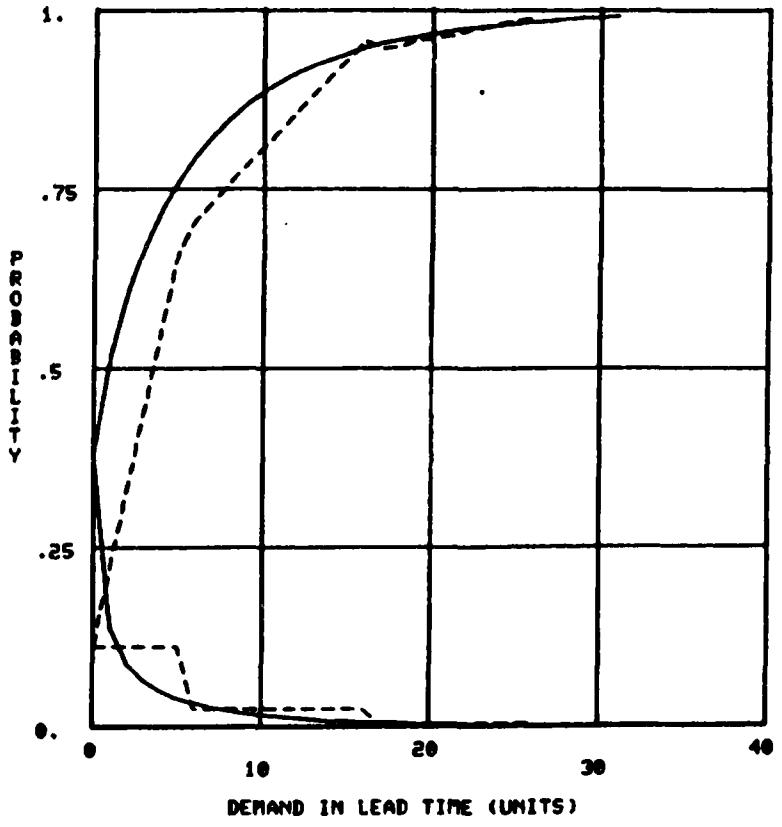
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	1.63
SKEWNESS	2.96
KURTOSIS	16.45

LPG PARAMETERS

THETA	0.90
LAMBDA	0.12
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.38	0.11	0.27
1.	0.52	0.22	0.30
2.	0.61	0.33	0.27
3.	0.67	0.44	0.23
4.	0.72	0.55	0.17
5.	0.76	0.66	0.10
6.	0.80	0.71	0.09
7.	0.82	0.73	0.09
8.	0.85	0.76	0.09
9.	0.87	0.78	0.08
10.	0.88	0.81	0.08
11.	0.90	0.83	0.07
12.	0.91	0.86	0.05
13.	0.92	0.88	0.04
14.	0.93	0.91	0.02
15.	0.94	0.93	0.01
16.	0.95	0.96	-0.01
17.	0.95	0.95	0.01
18.	0.96	0.95	0.01
19.	0.96	0.96	0.01
20.	0.97	0.96	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.296 FOR X = 1

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	8.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

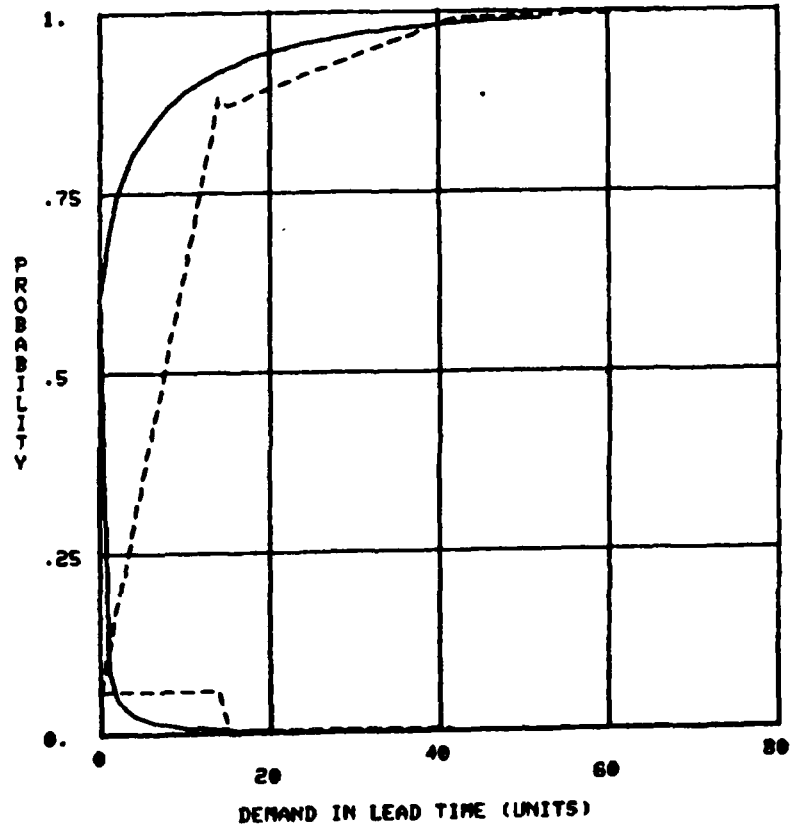
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	2.63
SKEWNESS	5.08
KURTOSIS	42.13

LPG PARAMETERS

THETA	0.96
LAMBDA	0.06
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.61	0.06	0.55
1.	0.70	0.12	0.58
2.	0.75	0.18	0.57
3.	0.78	0.23	0.54
4.	0.80	0.29	0.51
5.	0.82	0.35	0.47
6.	0.84	0.41	0.43
7.	0.86	0.47	0.39
8.	0.87	0.53	0.34
9.	0.88	0.59	0.29
10.	0.89	0.64	0.24
11.	0.90	0.70	0.19
12.	0.90	0.76	0.14
13.	0.91	0.82	0.09
14.	0.92	0.88	0.04
15.	0.92	0.87	0.05
16.	0.93	0.87	0.05
17.	0.93	0.88	0.05
18.	0.94	0.88	0.05
19.	0.94	0.89	0.05
20.	0.94	0.89	0.05



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.580 FOR X = 1

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	16.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

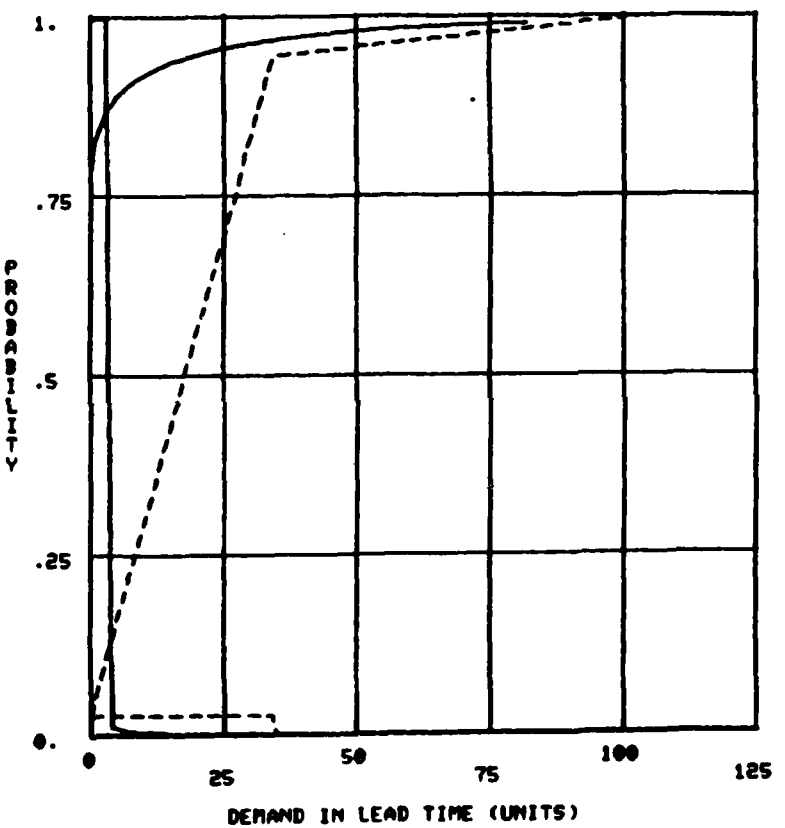
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	4.15
SKEWNESS	8.18
KURTOSIS	103.79

LPG PARAMETERS

THETA	0.99
LAMBDA	0.03
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.78	0.03	0.75
1.	0.83	0.05	0.77
2.	0.85	0.08	0.77
3.	0.86	0.11	0.76
4.	0.88	0.13	0.74
5.	0.89	0.16	0.73
6.	0.89	0.19	0.71
7.	0.90	0.21	0.69
8.	0.91	0.24	0.67
9.	0.91	0.27	0.64
10.	0.92	0.29	0.62
11.	0.92	0.32	0.60
12.	0.92	0.35	0.58
13.	0.93	0.37	0.55
14.	0.93	0.40	0.53
15.	0.93	0.43	0.51
16.	0.94	0.45	0.48
17.	0.94	0.48	0.46
18.	0.94	0.51	0.43
19.	0.94	0.53	0.41
20.	0.95	0.56	0.39



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.772 FOR X = 1

SAMPLE LPG CALCULATIONS

Data Set No.	6	7	8	9	10
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.50	.50	.50	.50	.50
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33
LPG Moments					
Mean	2.00	3.00	4.00	5.00	6.00
Skewness	2.16	1.72	1.46	1.28	1.14
Kurtosis	10.22	7.67	6.40	5.65	5.15

	6		7		8		9		10	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	1	2	2	3	3	4	4	5	5	6
.60	2	2	3	4	4	5	5	6	6	7
.70	2	4	4	5	5	6	6	7	8	8
.80	3	5	5	6	7	7	8	9	9	11
.85	4	5	6	7	8	8	9	9	11	12
.90	5	6	7	8	9	9	11	12	12	13
.95	7	8	10	9	12	12	14	14	15	15
.97	9	9	11	12	13	13	15	15	17	16
.99	12	9	15	14	17	16	19	19	22	20

Table II-3. Lead Time Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	4.00
C OF V OF LT	0.25

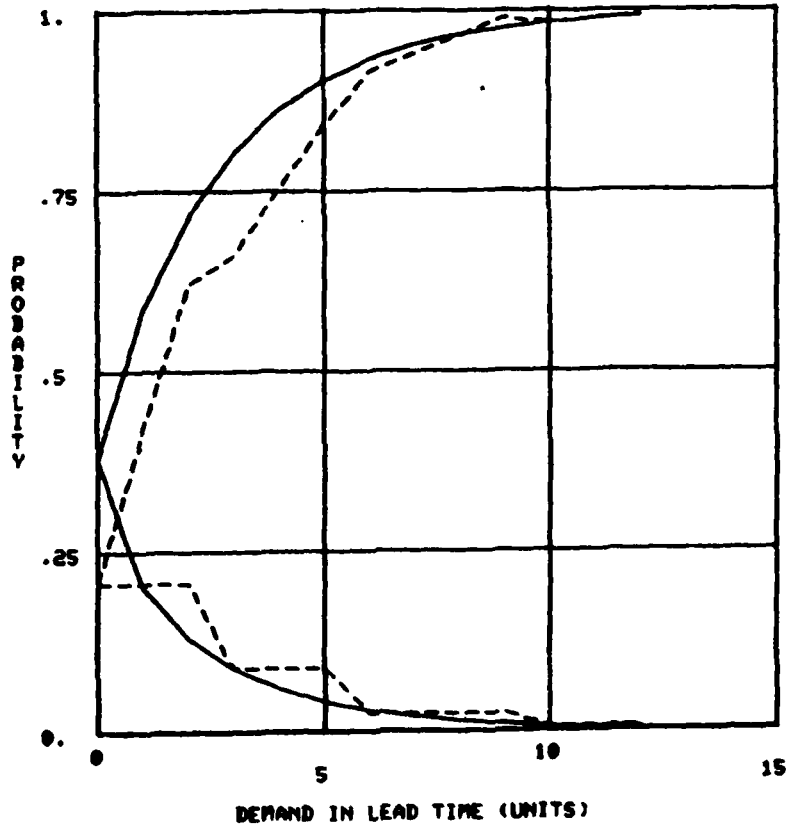
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	2.00
C OF V OF DLT	1.35
SKEWNESS	2.16
KURTOSIS	10.22

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	4.00

X	FX	FNBX	DIFF
0.	0.38	0.21	0.17
1.	0.58	0.41	0.17
2.	0.71	0.62	0.09
3.	0.80	0.66	0.14
4.	0.86	0.75	0.11
5.	0.90	0.84	0.06
6.	0.93	0.91	0.02
7.	0.95	0.94	0.01
8.	0.97	0.96	0.00
9.	0.98	0.99	-0.01
10.	0.98	0.98	-0.00
11.	0.99	0.99	-0.00
12.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.172 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	6.00
C OF V OF LT	0.25

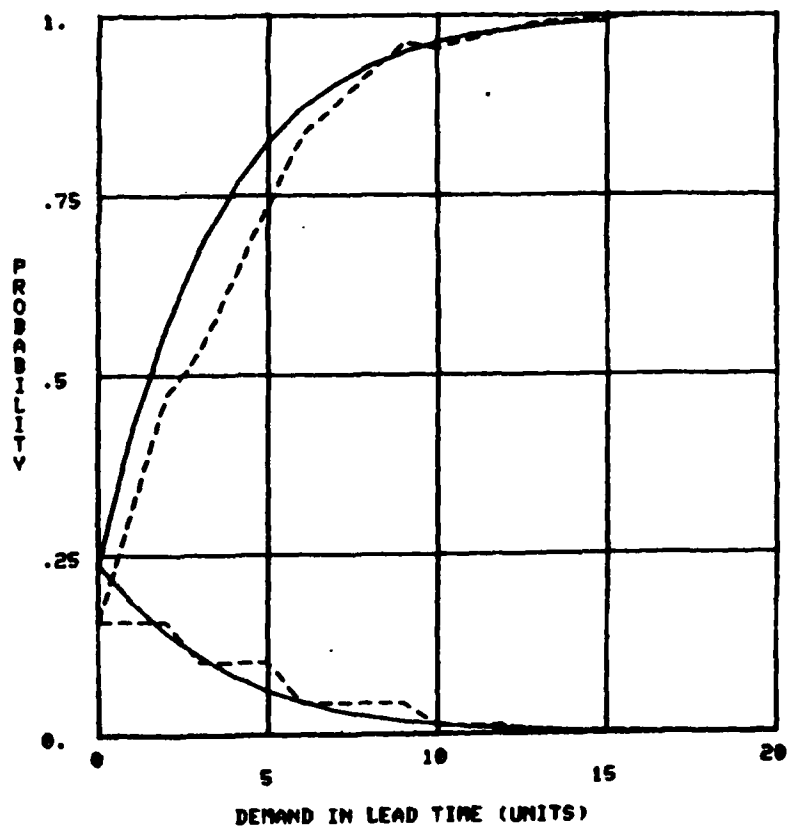
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	3.00
C OF V OF DLT	1.11
SKEWNESS	1.72
KURTOSIS	7.67

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.67

X	FX	FNBX	DIFF
0.	0.24	0.16	0.08
1.	0.42	0.32	0.11
2.	0.57	0.47	0.09
3.	0.68	0.54	0.14
4.	0.76	0.64	0.12
5.	0.82	0.74	0.09
6.	0.87	0.83	0.04
7.	0.90	0.87	0.03
8.	0.93	0.92	0.01
9.	0.95	0.96	-0.01
10.	0.96	0.95	0.01
11.	0.97	0.97	0.01
12.	0.98	0.98	0.00
13.	0.99	0.99	-0.00
14.	0.99	0.99	-0.00
15.	0.99	1.00	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.142 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

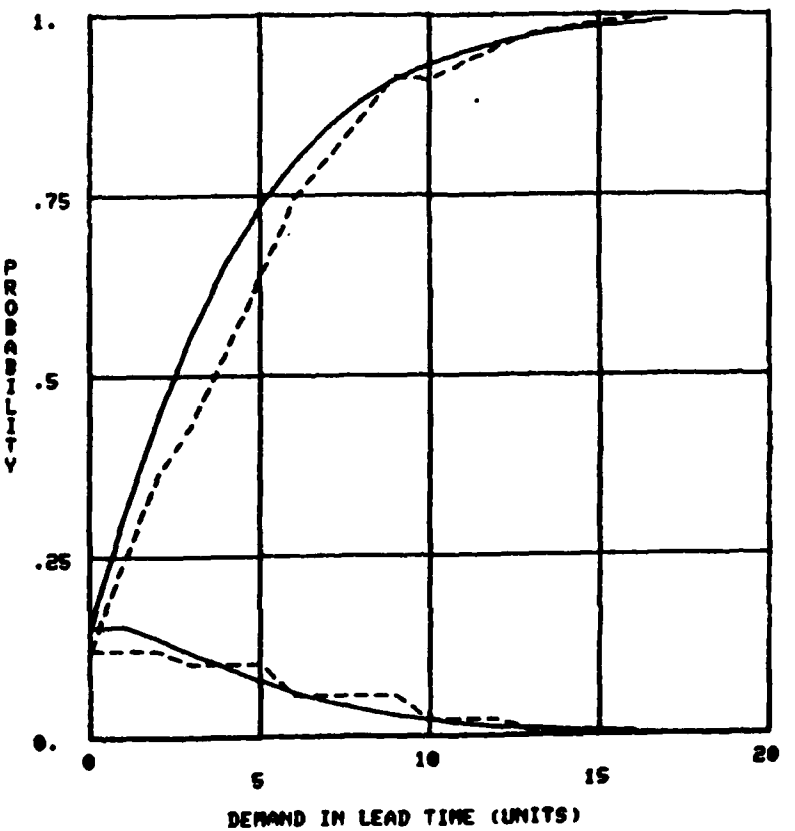
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	10.00
C OF V OF LT	0.25

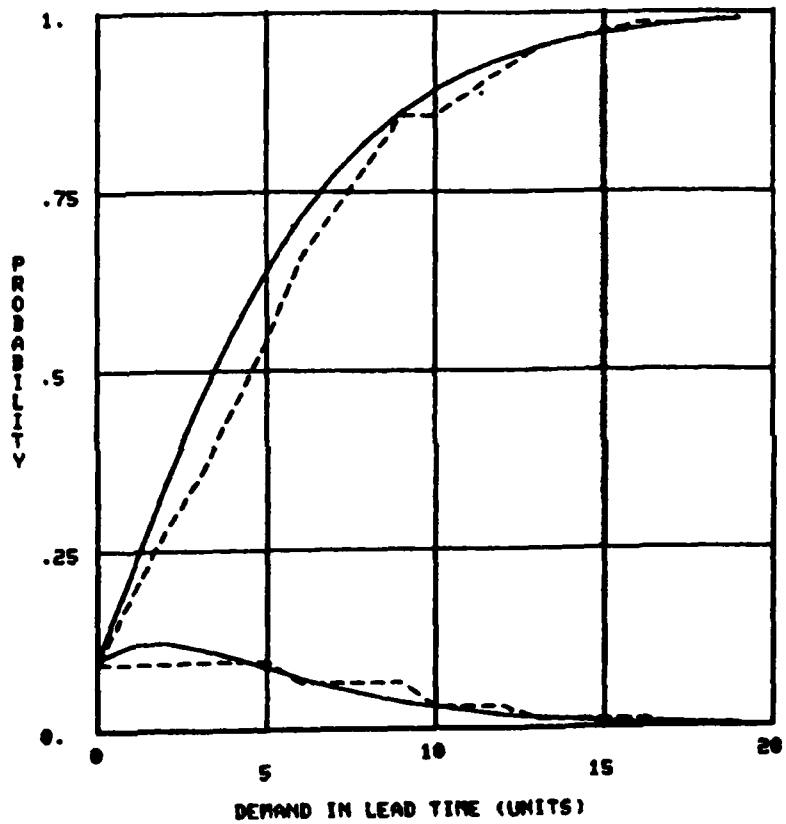
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	5.00
C OF V OF DLT	0.87
SKEWNESS	1.28
KURTOSIS	5.65

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	1.60

X	FX	FNBX	DIFF
0.	0.10	0.09	0.01
1.	0.22	0.19	0.03
2.	0.34	0.28	0.06
3.	0.45	0.35	0.10
4.	0.55	0.45	0.11
5.	0.64	0.54	0.10
6.	0.71	0.66	0.06
7.	0.77	0.72	0.05
8.	0.82	0.79	0.03
9.	0.86	0.85	0.01
10.	0.89	0.85	0.04
11.	0.92	0.89	0.03
12.	0.93	0.92	0.02
13.	0.95	0.95	0.00
14.	0.96	0.96	0.00
15.	0.97	0.97	-0.00
16.	0.98	0.99	-0.01
17.	0.98	0.98	-0.00
18.	0.99	0.99	-0.00
19.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.108 FOR X = 4

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	12.00
C OF V OF LT	0.25

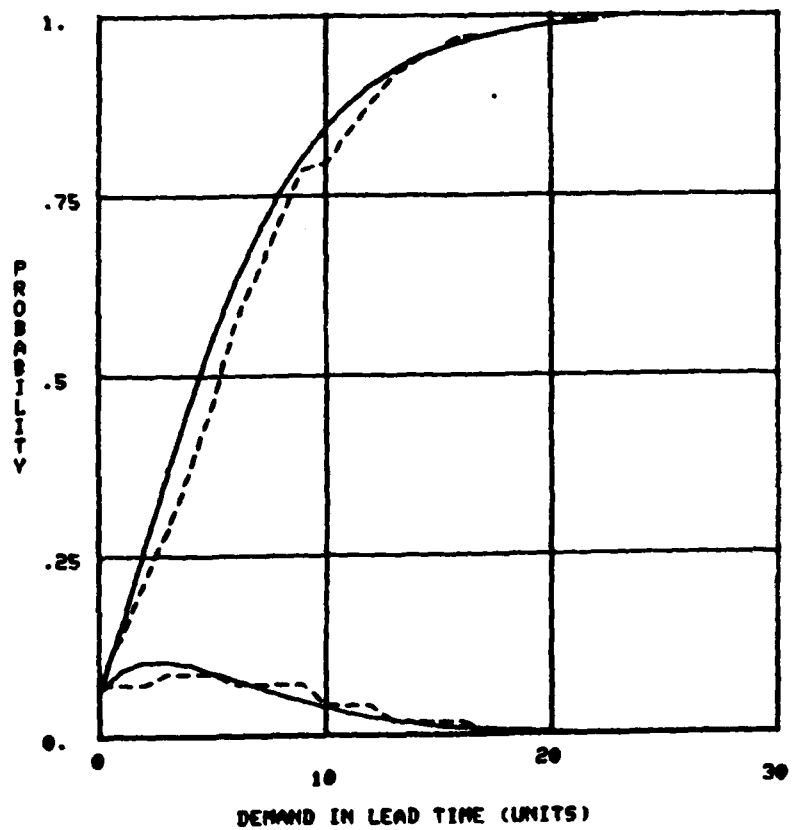
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	6.00
C OF V OF DLT	0.81
SKEWNESS	1.14
KURTOSIS	5.15

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	1.33

X	FX	FNBX	DIFF
0.	0.06	0.07	-0.01
1.	0.16	0.14	0.01
2.	0.26	0.21	0.04
3.	0.36	0.28	0.08
4.	0.46	0.37	0.09
5.	0.55	0.46	0.09
6.	0.63	0.57	0.06
7.	0.70	0.64	0.05
8.	0.75	0.72	0.04
9.	0.80	0.79	0.02
10.	0.84	0.79	0.05
11.	0.87	0.83	0.04
12.	0.90	0.87	0.03
13.	0.92	0.91	0.01
14.	0.94	0.93	0.00
15.	0.95	0.95	-0.00
16.	0.96	0.97	-0.01
17.	0.97	0.97	0.00
18.	0.98	0.98	0.00
19.	0.98	0.98	-0.00
20.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.093 FOR X = 5

SAMPLE LPG CALCULATIONS

Data Set No.	11	12	13	14	15
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosis	21.73	6.40	4.53	3.93	3.64

	11		12		13		14		15	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	26	26
.90	3	5	9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	13	21	21	28	28	35	35
.99	8	8	17	16	26	25	34	33	41	40

Table II-4. Demand Rate Sensitivity for LO Base Case

EMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.10
AVE LEAD TIME	5.00
C OF V OF LT	0.25

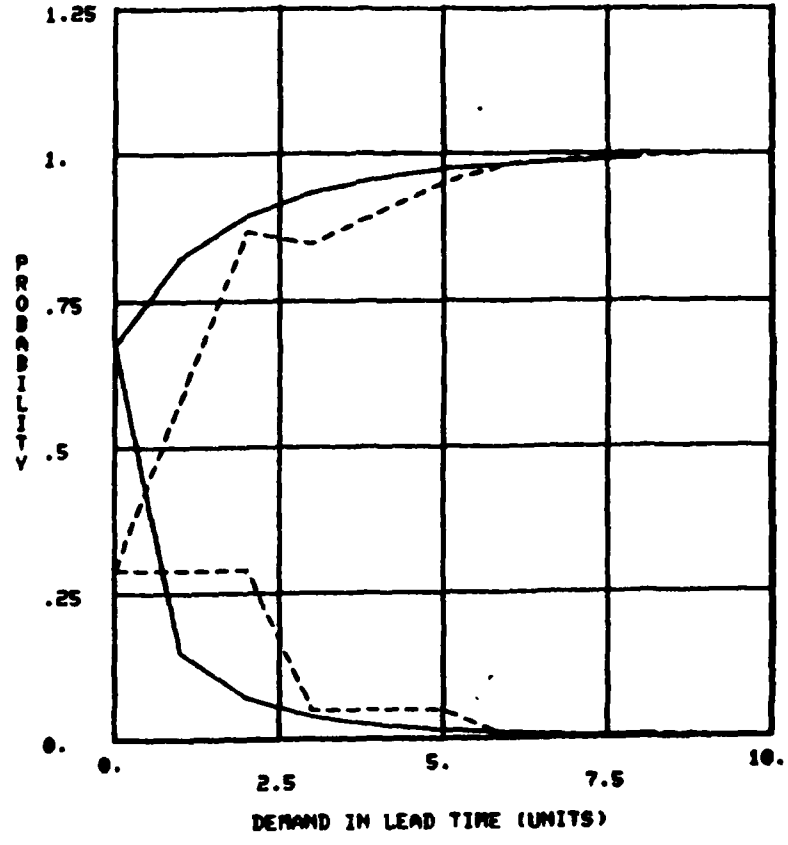
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	0.80
C OF V OF DLT	2.11
SKEWNESS	3.52
KURTOSIS	21.73

LPG PARAMETERS

THETA	0.72
LAMBDA	0.05
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.67	0.29	0.38
1.	0.82	0.58	0.24
2.	0.89	0.87	0.03
3.	0.93	0.85	0.09
4.	0.96	0.90	0.06
5.	0.97	0.95	0.02
6.	0.98	0.98	-0.00
7.	0.99	0.99	-0.00
8.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.384 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

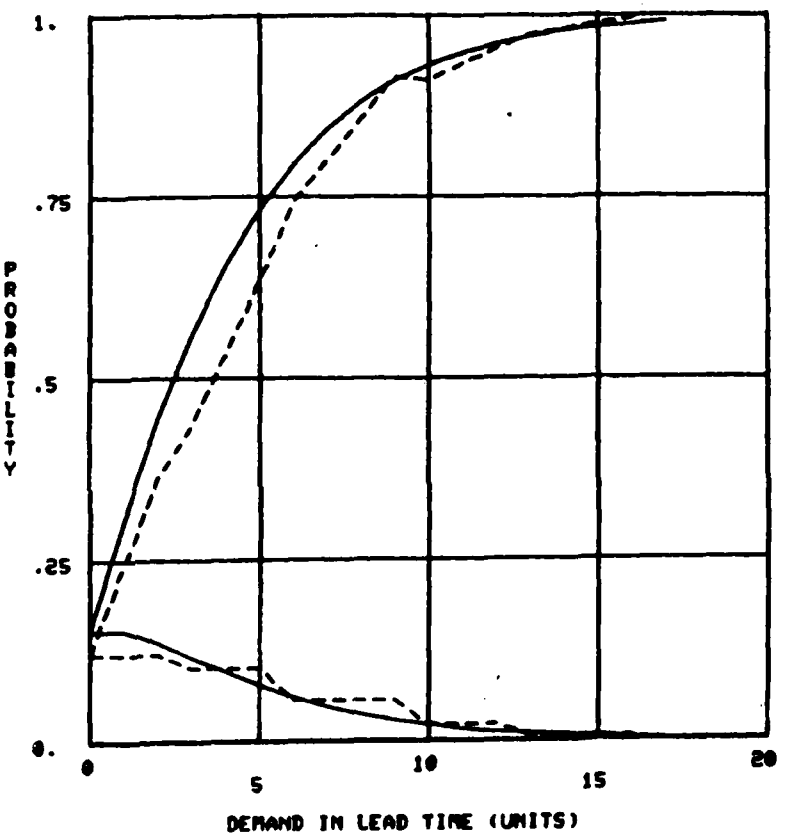
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	1.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

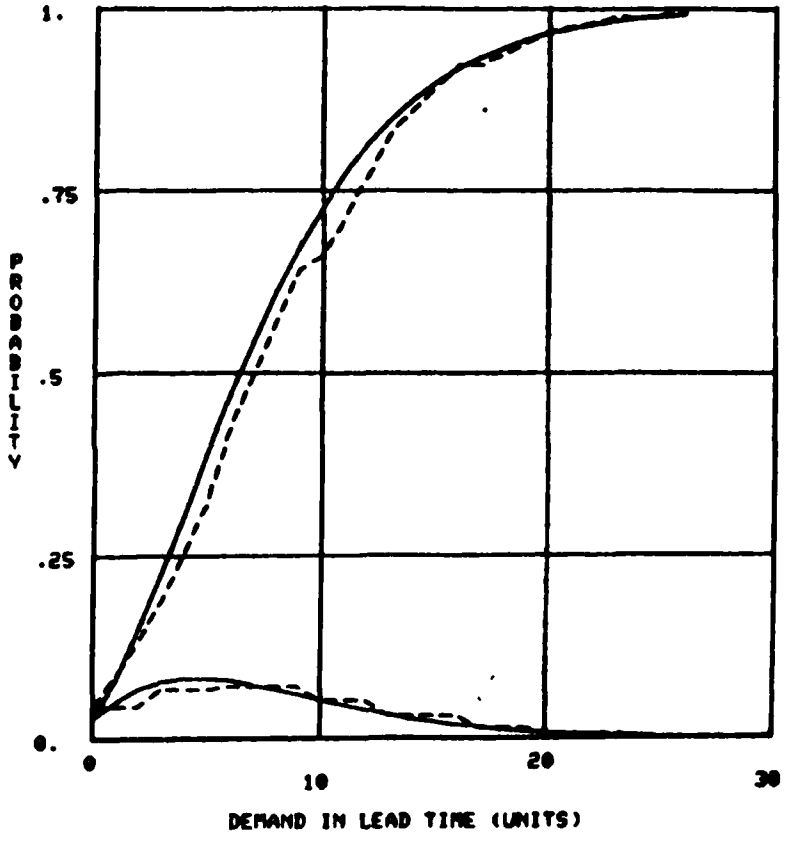
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	8.00
C OF V OF DLT	0.71
SKEWNESS	0.95
KURTOSIS	4.53

LPG PARAMETERS

THETA	0.72
LAMBDA	0.50
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.73	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.89	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.96	0.95	0.00
20.	0.97	0.97	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	1.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

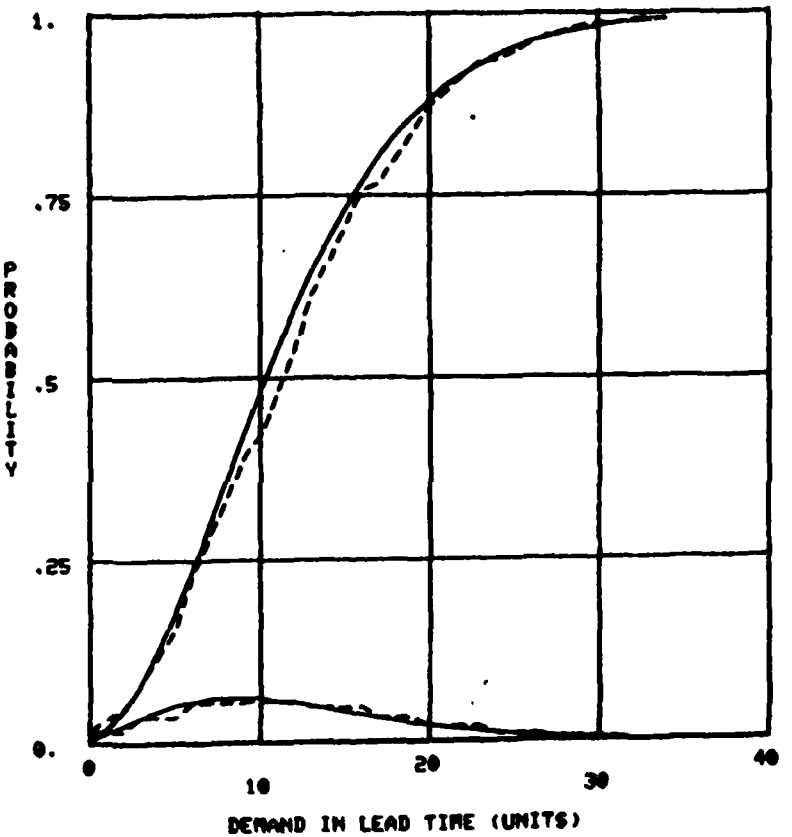
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	12.00
C OF V OF DLT	0.60
SKEWNESS	0.73
KURTOSIS	3.92

LPG PARAMETERS

THETA	0.72
LAMBDA	0.75
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.01
1.	0.02	0.03	-0.01
2.	0.05	0.05	-0.00
3.	0.08	0.08	0.00
4.	0.13	0.12	0.01
5.	0.18	0.15	0.03
6.	0.24	0.23	0.01
7.	0.30	0.28	0.02
8.	0.36	0.34	0.02
9.	0.42	0.39	0.03
10.	0.48	0.42	0.06
11.	0.54	0.48	0.06
12.	0.59	0.53	0.06
13.	0.64	0.61	0.03
14.	0.69	0.66	0.03
15.	0.73	0.71	0.02
16.	0.77	0.76	0.01
17.	0.80	0.77	0.04
18.	0.83	0.80	0.03
19.	0.86	0.83	0.02
20.	0.88	0.87	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.062 FOR X = 11

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

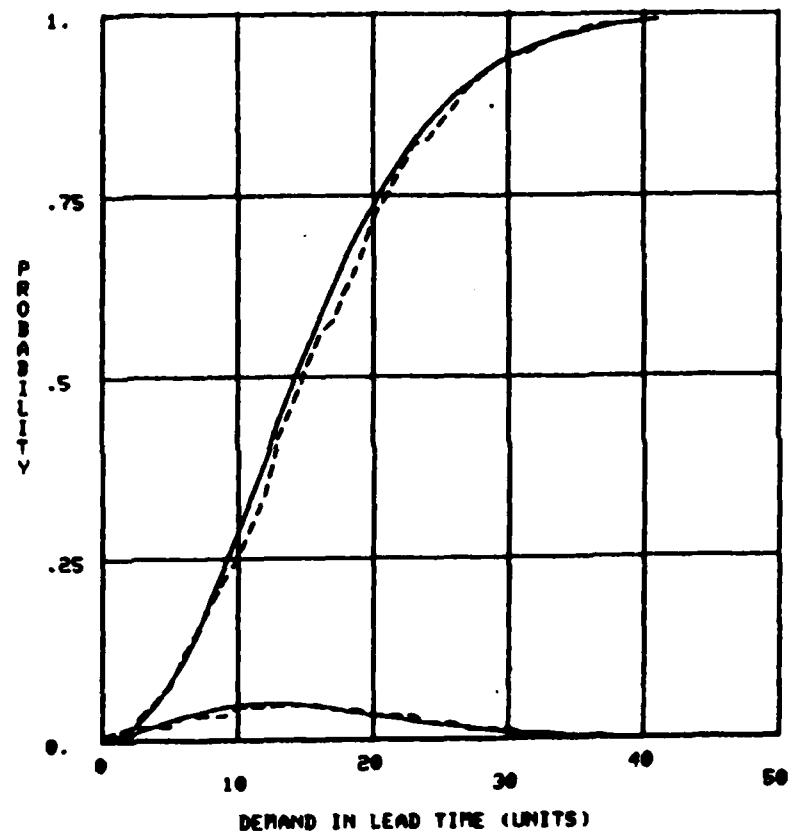
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

SAMPLE LPG CALCULATION

Data Set No.	16	17	18	19	20
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	0.5	0.5	0.5	0.5	0.5
Mean Lead Time	8	8.0	8.0	8.0	8.0
CV of Lead Time	.01	.25	.50	.75	1.0
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.25	.25	.25	.25	.25
Alpha	1,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13
LPG Moments					
Mean	4.00	4.00	4.00	4.00	4.00
CV	.94	.97	1.06	1.20	1.37
Skewness	1.61	1.46	1.21	1.13	1.29
Kurtosis	6.84	6.40	5.55	5.15	5.76

	16		17		18		19		20	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	3	4	3	4	3	4	2	3	2	2
.60	4	5	4	5	4	5	3	5	3	4
.70	5	6	5	6	5	6	5	6	5	6
.80	7	7	7	7	7	8	7	8	7	8
.85	8	8	8	8	8	9	8	9	9	9
.90	9	9	9	9	10	11	10	11	11	12
.95	11	12	12	12	12	13	14	14	15	15
.97	13	13	13	13	14	14	16	17	18	19
.99	17	15	17	16	19	19	21	21	25	25

Table II-5. Lead Time Variability Sensitivity for LO Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.01

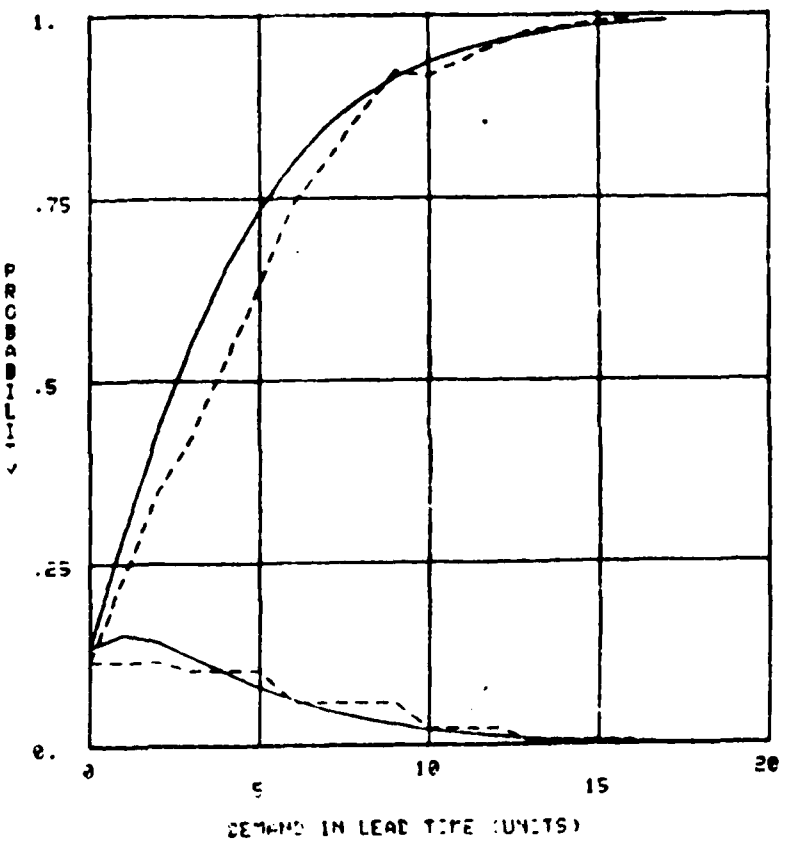
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DL	0.94
SKEWNESS	1.61
KURTOSIS	6.84

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	10000.00
BETA	1250.00

X	FX	FNBX	DIFF
0.	0.14	0.12	0.02
1.	0.29	0.23	0.06
2.	0.43	0.35	0.08
3.	0.55	0.42	0.13
4.	0.66	0.53	0.13
5.	0.74	0.63	0.10
6.	0.80	0.74	0.05
7.	0.85	0.80	0.04
8.	0.89	0.86	0.02
9.	0.91	0.92	-0.01
10.	0.94	0.93	0.02
11.	0.95	0.94	0.01
12.	0.96	0.96	0.00
13.	0.97	0.98	-0.00
14.	0.98	0.98	-0.00
15.	0.99	0.99	-0.00
16.	0.99	1.00	-0.01



[illegible] OF LPG PROBABILITIES
[illegible] NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.13: FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

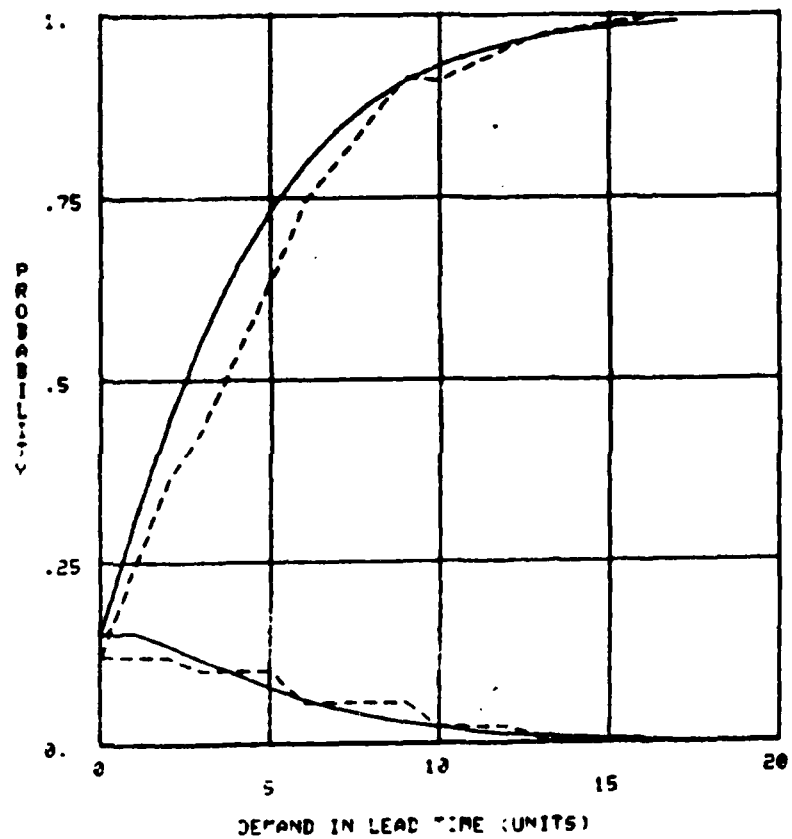
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBx	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.50

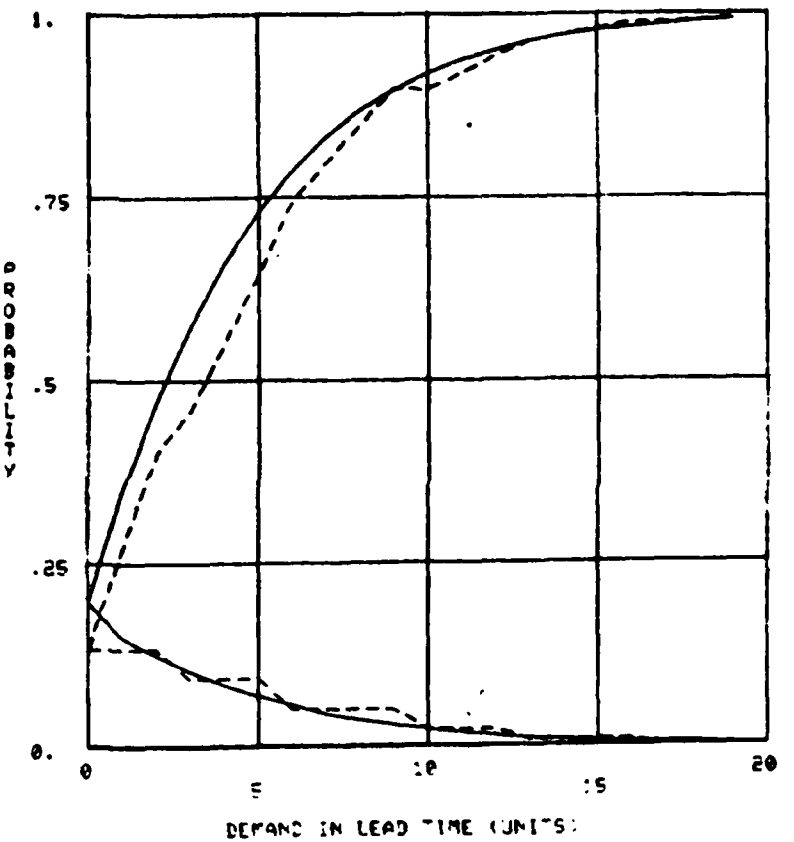
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	1.06
SKEWNESS	1.21
KURTOSIS	5.55

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	4.00
BETA	0.50

X	FX	FNBX	DIFF
0.	0.20	0.13	0.06
1.	0.35	0.27	0.08
2.	0.47	0.40	0.07
3.	0.58	0.46	0.12
4.	0.66	0.55	0.11
5.	0.73	0.65	0.08
6.	0.79	0.74	0.04
7.	0.83	0.80	0.04
8.	0.87	0.85	0.02
9.	0.90	0.90	-0.00
10.	0.92	0.90	0.02
11.	0.94	0.92	0.02
12.	0.95	0.94	0.01
13.	0.96	0.96	-0.00
14.	0.97	0.97	-0.00
15.	0.98	0.98	-0.00
16.	0.98	0.99	-0.01
17.	0.99	0.99	-0.00
18.	0.99	0.99	-0.00
19.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.116 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	3.00
C OF V OF LT	0.75

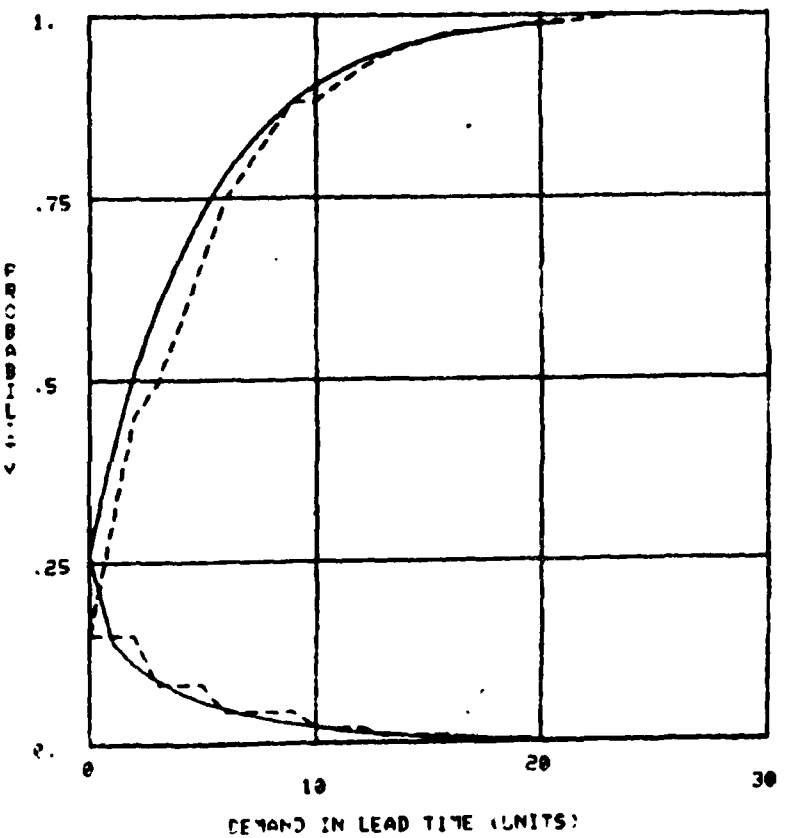
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	1.20
SKEWNESS	1.13
KURTOSIS	5.15

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	1.78
BETA	0.22

X	FX	FNBX	DIFF
0.	0.26	0.15	0.11
1.	0.40	0.30	0.10
2.	0.51	0.45	0.06
3.	0.60	0.50	0.10
4.	0.67	0.58	0.09
5.	0.73	0.66	0.07
6.	0.78	0.75	0.03
7.	0.82	0.79	0.03
8.	0.85	0.84	0.02
9.	0.88	0.88	0.00
10.	0.90	0.88	0.02
11.	0.92	0.90	0.02
12.	0.94	0.92	0.01
13.	0.95	0.95	0.00
14.	0.96	0.96	0.00
15.	0.97	0.97	-0.00
16.	0.97	0.98	-0.00
17.	0.98	0.98	0.00
18.	0.98	0.98	0.00
19.	0.99	0.99	0.00
20.	0.99	0.99	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX DIF DIFFERENCE= 0.111 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	1.00

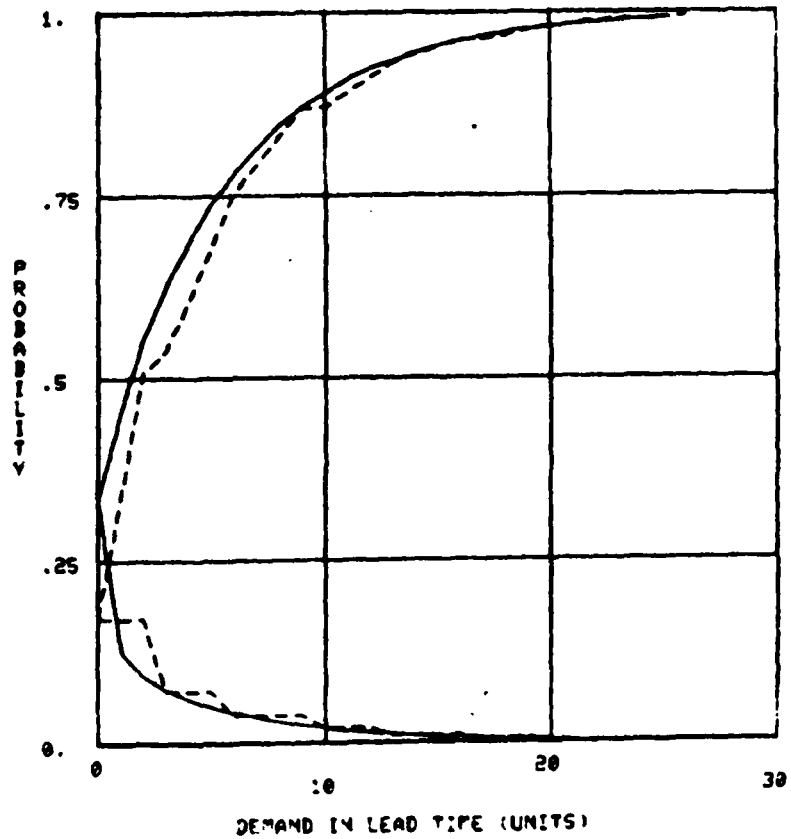
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	1.37
SKEWNESS	1.29
KURTOSIS	5.76

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	1.00
BETA	0.13

X	Fx	FNBX	DIFF
0.	0.33	0.17	0.16
1.	0.46	0.34	0.12
2.	0.55	0.51	0.04
3.	0.63	0.54	0.09
4.	0.69	0.61	0.08
5.	0.74	0.68	0.06
6.	0.78	0.75	0.03
7.	0.81	0.79	0.02
8.	0.84	0.83	0.01
9.	0.87	0.87	0.00
10.	0.89	0.87	0.02
11.	0.91	0.89	0.02
12.	0.92	0.71	0.21
13.	0.93	0.93	0.00
14.	0.94	0.94	0.00
15.	0.95	0.95	0.00
16.	0.96	0.96	-0.00
17.	0.97	0.96	0.00
18.	0.97	0.97	0.00
19.	0.98	0.97	0.00
20.	0.98	0.98	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

Max CDF DIFFERENCE= 0.164 FOR X = 0

SAMPLE LPG CALCULATIONS

Data Set No.	21	22	23	24	25
Ave. Req. Size	1.01	2.00	4.00	8.00	16.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.02	.72	.90	.96	.99
Lambda	1.98	1.00	.50	.25	.13
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	16.00	16.00	16.00	16.00	16.00
CV	.36	.53	.84	1.33	2.09
Skewness	.27	.60	1.35	2.44	4.02
Kurtosis	3.13	3.64	5.97	12.32	27.79

Percentage Points	21		22		23		24		25	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	15	16	15	15	13	14	8	13	2	22
.60	17	18	17	18	16	19	12	18	5	26
.70	19	19	19	20	20	23	18	26	11	31
.80	21	21	23	23	25	27	27	35	23	37
.85	22	22	25	26	29	31	33	39	33	56
.90	24	24	27	28	34	35	43	47	50	75
.95	26	27	32	32	42	42	59	61	80	93
.97	28	28	35	34	49	46	72	67	105	101
.99	31	32	41	40	61	57	99	88	163	148

Table II-6. Requisition Size Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	1.01
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

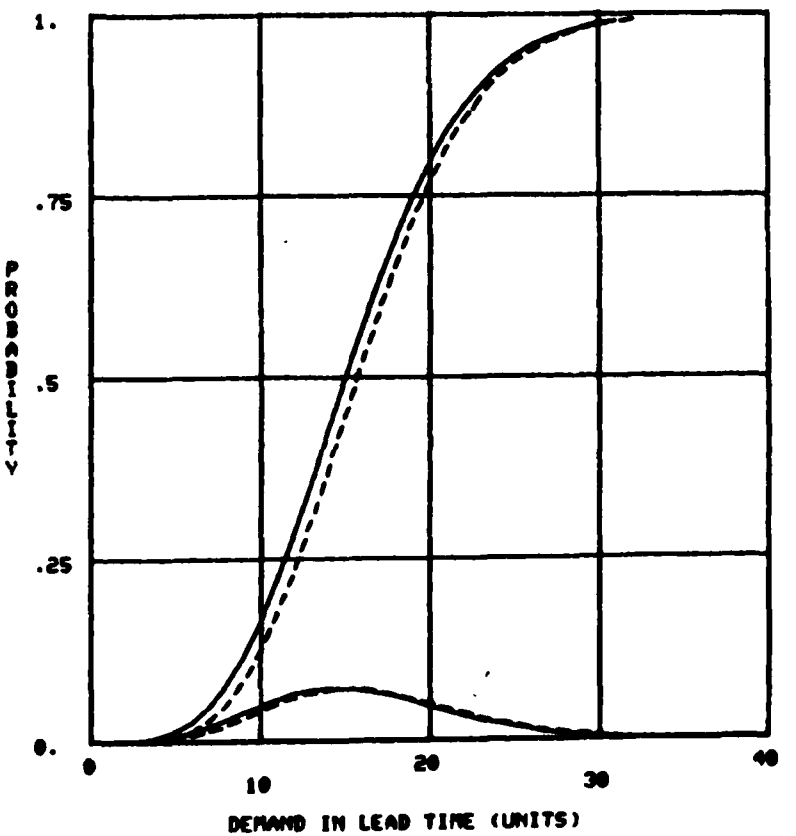
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.36
SKEWNESS	0.27
KURTOSIS	3.13

LPG PARAMETERS

THETA	0.02
LAMBDA	1.98
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.00	0.00
1.	0.00	0.00	0.00
2.	0.00	0.00	0.00
3.	0.00	0.00	0.00
4.	0.01	0.00	0.00
5.	0.01	0.01	0.01
6.	0.03	0.01	0.01
7.	0.05	0.03	0.02
8.	0.08	0.05	0.03
9.	0.12	0.08	0.03
10.	0.16	0.12	0.04
11.	0.22	0.18	0.05
12.	0.29	0.24	0.05
13.	0.36	0.30	0.05
14.	0.43	0.38	0.05
15.	0.50	0.45	0.06
16.	0.57	0.52	0.05
17.	0.64	0.59	0.04
18.	0.70	0.66	0.04
19.	0.75	0.72	0.03
20.	0.80	0.77	0.03



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.053 FOR X = 14

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

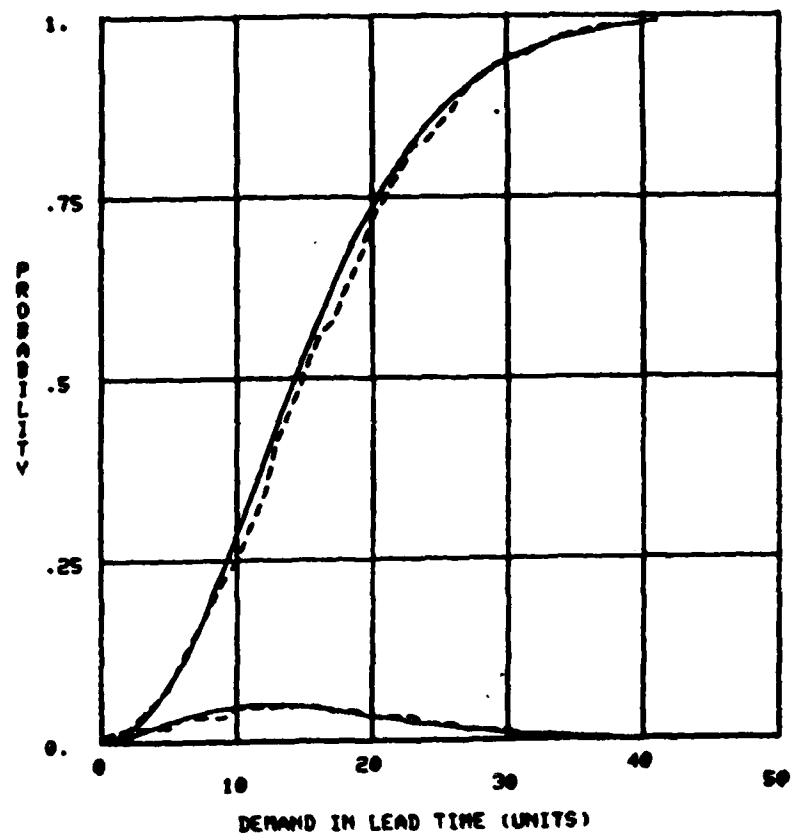
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	4.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

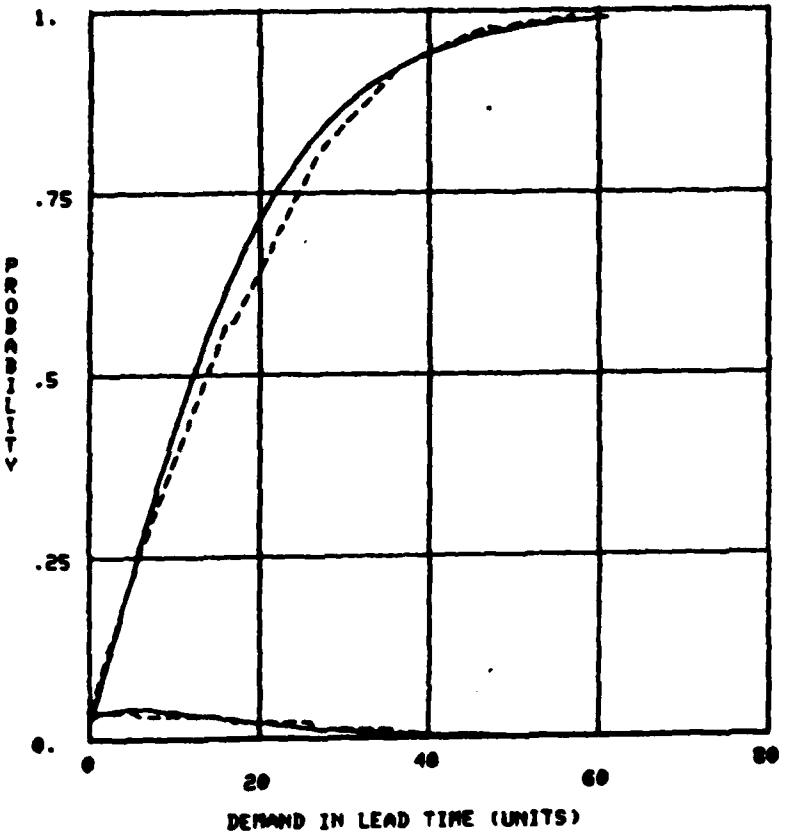
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.84
SKEWNESS	1.35
KURTOSIS	5.97

LPG PARAMETERS

THETA	0.90
LAMBDA	0.50
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.01
1.	0.06	0.07	-0.01
2.	0.10	0.11	-0.01
3.	0.14	0.15	-0.01
4.	0.18	0.19	-0.00
5.	0.23	0.22	0.00
6.	0.27	0.26	0.01
7.	0.31	0.29	0.02
8.	0.35	0.32	0.03
9.	0.39	0.35	0.04
10.	0.43	0.38	0.04
11.	0.46	0.42	0.05
12.	0.50	0.45	0.05
13.	0.53	0.48	0.05
14.	0.56	0.51	0.05
15.	0.59	0.54	0.05
16.	0.62	0.57	0.05
17.	0.64	0.57	0.07
18.	0.67	0.60	0.07
19.	0.69	0.62	0.07
20.	0.71	0.64	0.07



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.071 FOR X = 18

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	8.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

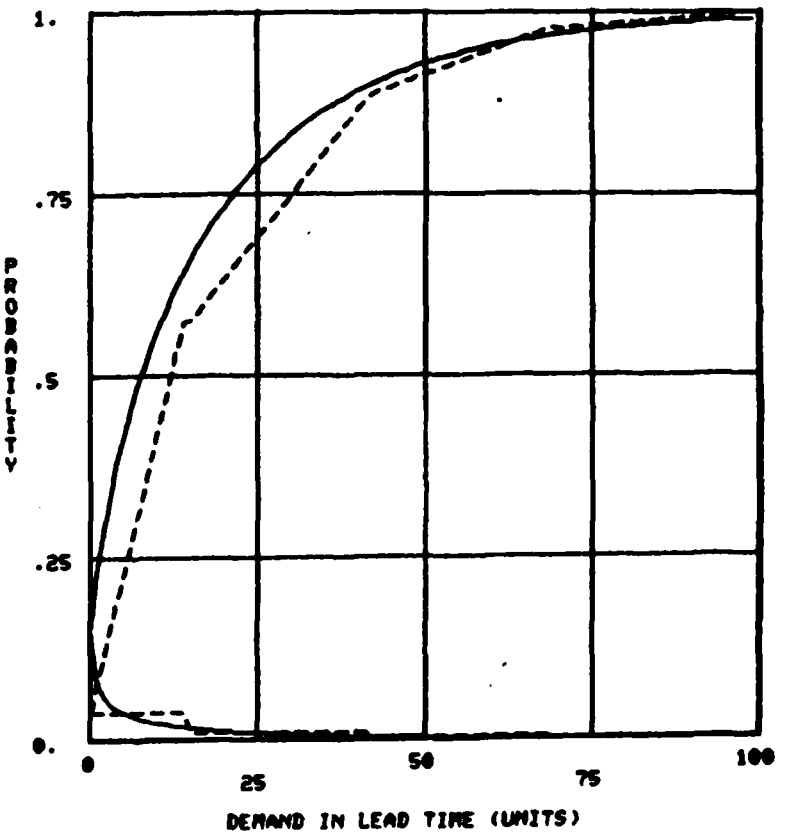
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	1.33
SKEWNESS	2.44
KURTOSIS	12.32

LPG PARAMETERS

THETA	0.96
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.04	0.11
1.	0.23	0.08	0.15
2.	0.29	0.11	0.17
3.	0.34	0.15	0.19
4.	0.38	0.19	0.19
5.	0.42	0.23	0.19
6.	0.45	0.27	0.19
7.	0.48	0.31	0.18
8.	0.51	0.34	0.17
9.	0.54	0.38	0.16
10.	0.56	0.42	0.14
11.	0.58	0.46	0.13
12.	0.61	0.50	0.11
13.	0.63	0.54	0.09
14.	0.64	0.57	0.07
15.	0.66	0.58	0.08
16.	0.68	0.59	0.09
17.	0.69	0.60	0.09
18.	0.71	0.61	0.10
19.	0.72	0.62	0.10
20.	0.73	0.63	0.10



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.190 FOR X = 4

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	16.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

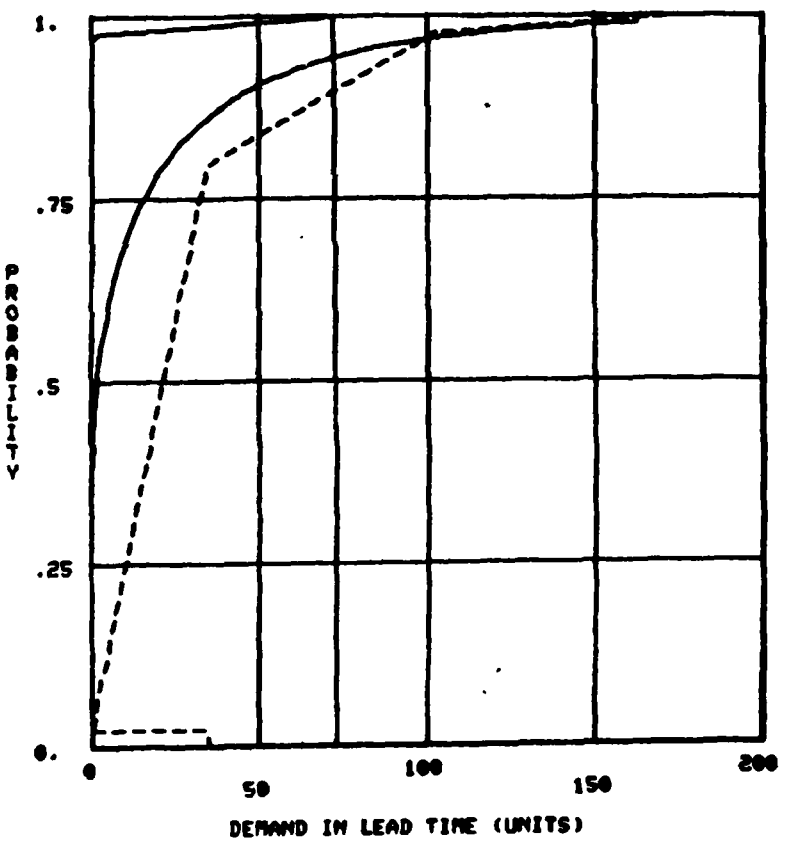
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	2.09
SKEWNESS	4.02
KURTOSIS	27.70

LPG PARAMETERS

THETA	0.99
LAMBDA	0.13
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.38	0.02	0.36
1.	0.46	0.04	0.42
2.	0.51	0.07	0.45
3.	0.55	0.09	0.46
4.	0.58	0.11	0.47
5.	0.60	0.13	0.47
6.	0.63	0.16	0.47
7.	0.64	0.18	0.47
8.	0.66	0.20	0.46
9.	0.68	0.22	0.45
10.	0.69	0.25	0.44
11.	0.70	0.27	0.43
12.	0.71	0.29	0.42
13.	0.73	0.31	0.41
14.	0.74	0.34	0.40
15.	0.75	0.36	0.39
16.	0.75	0.38	0.37
17.	0.76	0.40	0.36
18.	0.77	0.43	0.34
19.	0.78	0.45	0.33
20.	0.79	0.47	0.31



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.470 FOR X = 5

SAMPLE LPG CALCULATIONS

Data Set No.	26	27	28	29	30
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	4.00	6.00	8.00	10.00	12.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	4.00	2.67	2.00	1.60	1.33
LPG Moments					
Mean	8.00	12.00	16.00	20.00	24.00
CV	.71	.60	.53	.49	.46
Skewness	.95	.73	.60	.52	.47
Kurtosis	4.53	3.92	3.64	3.48	3.38

	26		27		28		29		30	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	7	8	11	12	15	15	19	20	23	23
.60	8	9	13	13	17	18	21	22	25	26
.70	10	11	15	15	19	20	24	25	29	29
.80	12	13	17	19	23	23	28	28	33	33
.85	14	14	19	20	25	26	30	30	35	36
.90	16	16	22	22	27	28	33	34	39	39
.95	19	19	25	26	32	32	38	37	44	44
.97	21	21	28	28	35	34	41	41	48	48
.99	26	25	34	33	41	40	48	48	55	55

Table II-7. Lead Time Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	4.00
C OF V OF LT	0.25

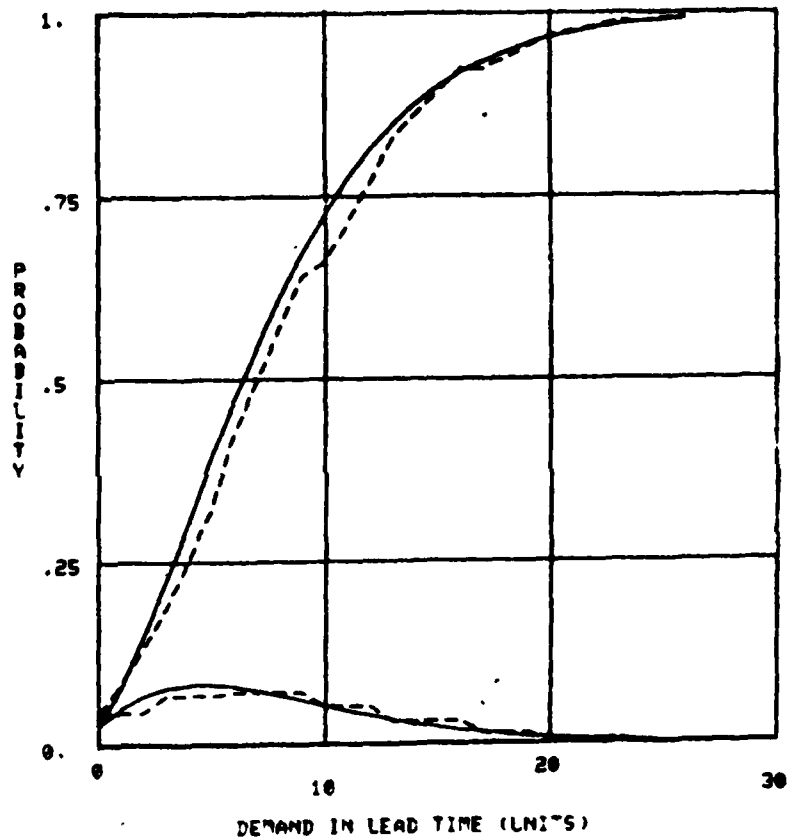
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	8.00
C OF V OF DLT	0.71
SKEWNESS	0.95
KURTOSIS	4.53

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	4.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.73	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.89	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.96	0.95	0.00
20.	0.97	0.97	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	6.00
C OF V OF LT	0.25

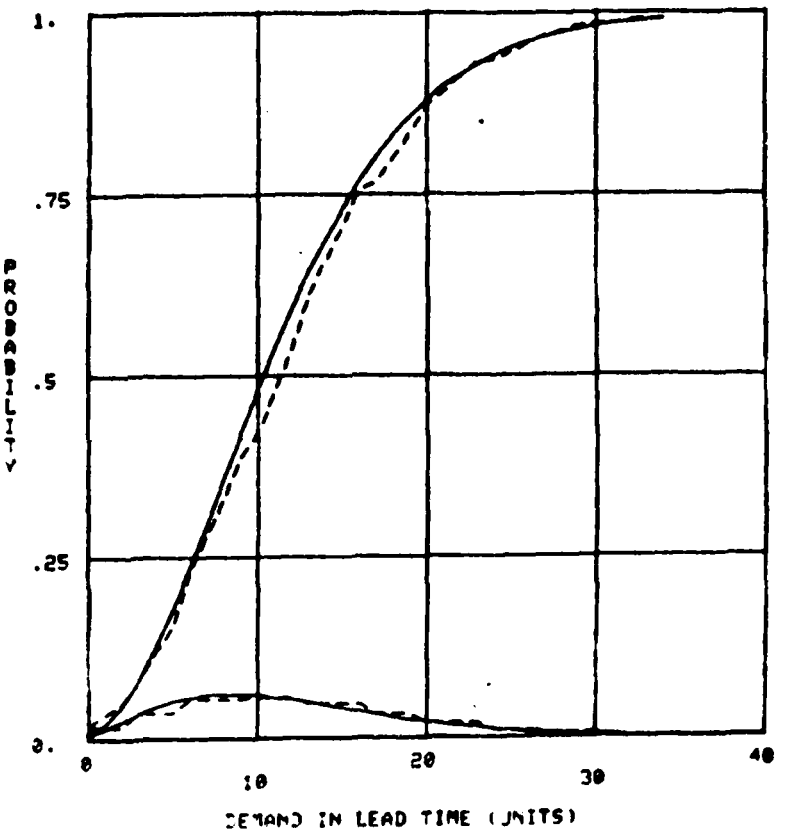
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	12.00
C OF V OF DLT	0.60
SKEWNESS	0.73
KURTOSIS	3.92

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.67

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.01
1.	0.02	0.03	-0.01
2.	0.05	0.05	-0.00
3.	0.08	0.08	0.00
4.	0.13	0.12	0.01
5.	0.18	0.15	0.03
6.	0.24	0.23	0.01
7.	0.30	0.28	0.02
8.	0.36	0.34	0.02
9.	0.42	0.39	0.03
10.	0.48	0.42	0.06
11.	0.54	0.48	0.06
12.	0.59	0.53	0.06
13.	0.64	0.61	0.03
14.	0.69	0.66	0.03
15.	0.73	0.71	0.02
16.	0.77	0.76	0.01
17.	0.80	0.77	0.04
18.	0.83	0.80	0.03
19.	0.86	0.83	0.02
20.	0.88	0.87	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.062 FOR X = 11

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

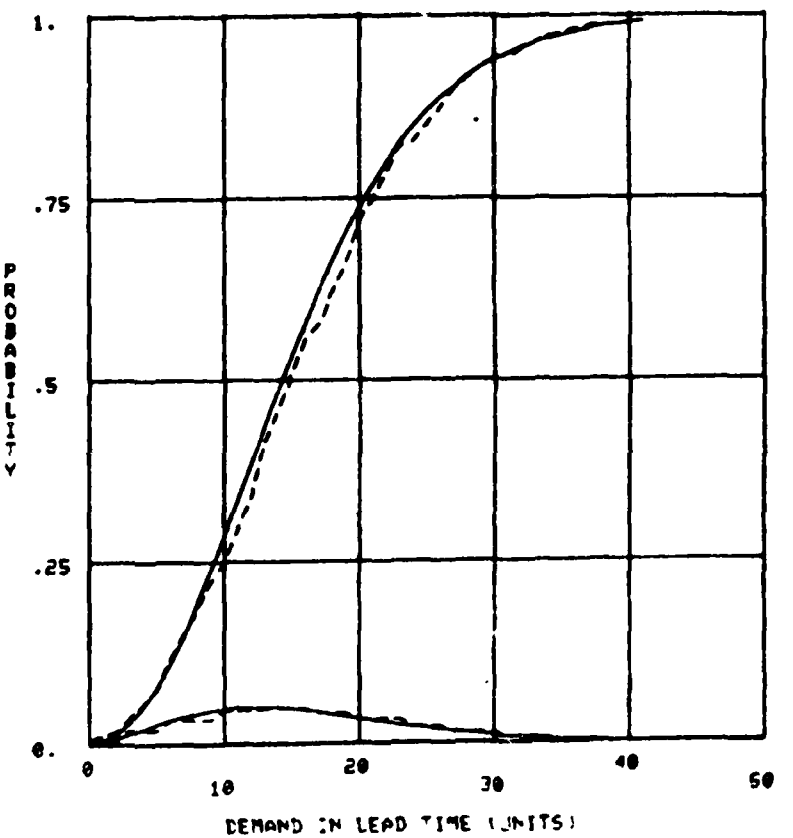
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.59	0.56	0.02
17.	0.62	0.59	0.05
18.	0.67	0.63	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

(-1

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	10.00
C OF V OF LT	0.25

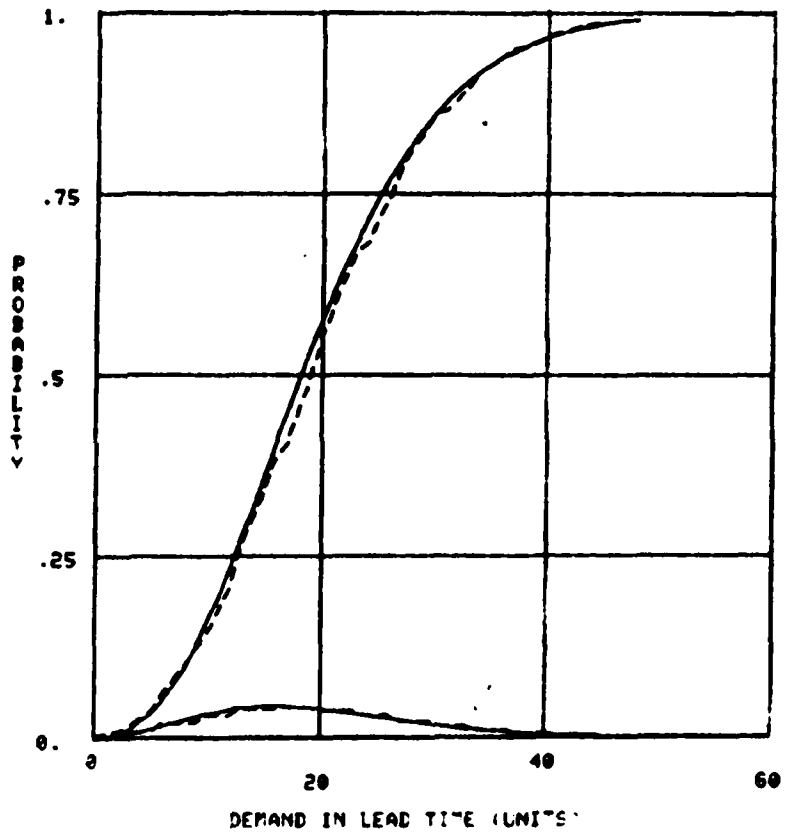
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	20.00
C OF V OF DLT	0.49
SKEWNESS	0.52
KURTOSIS	3.48

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	1.60

X	FX	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.01	-0.00
2.	0.01	0.01	-0.00
3.	0.01	0.02	-0.01
4.	0.02	0.03	-0.01
5.	0.03	0.04	-0.00
6.	0.05	0.06	-0.01
7.	0.07	0.08	-0.01
8.	0.10	0.10	-0.00
9.	0.13	0.12	0.00
10.	0.16	0.14	0.02
11.	0.20	0.18	0.02
12.	0.24	0.21	0.03
13.	0.28	0.27	0.01
14.	0.32	0.31	0.01
15.	0.36	0.35	0.02
16.	0.41	0.39	0.02
17.	0.45	0.41	0.04
18.	0.49	0.45	0.04
19.	0.53	0.49	0.04
20.	0.57	0.55	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.042 FOR X = 18

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	12.00
C OF V OF LT	0.25

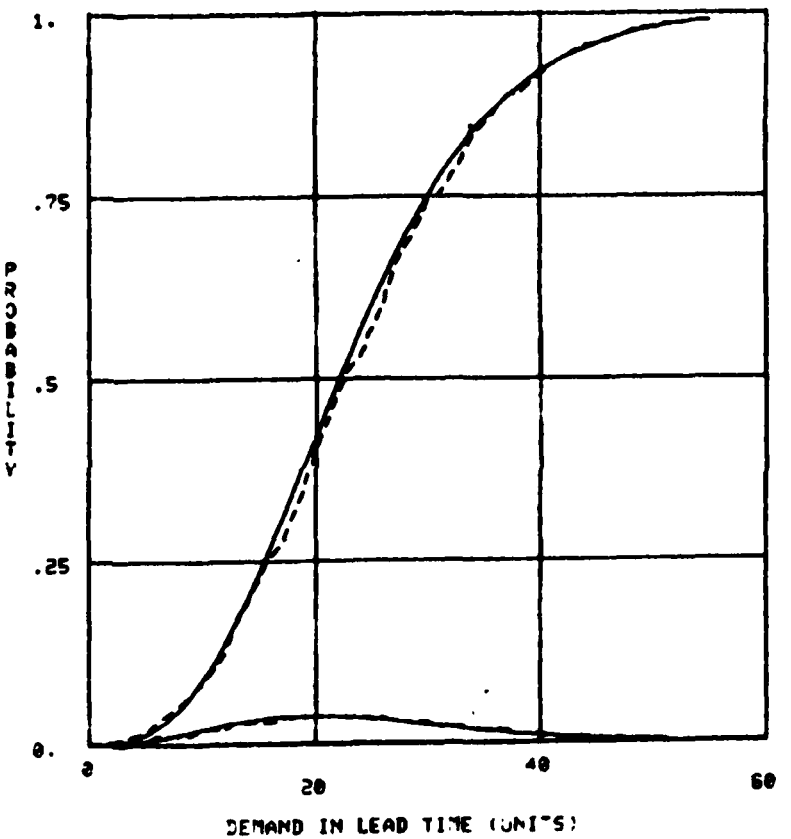
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	24.00
C OF V OF DLT	0.46
SKEWNESS	0.47
KURTOSIS	3.38

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	1.33

X	FX	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.00	-0.00
2.	0.00	0.00	-0.00
3.	0.00	0.01	-0.00
4.	0.01	0.01	-0.00
5.	0.01	0.02	-0.00
6.	0.02	0.03	-0.01
7.	0.03	0.04	-0.01
8.	0.05	0.05	-0.01
9.	0.07	0.07	-0.00
10.	0.09	0.08	0.00
11.	0.11	0.10	0.01
12.	0.14	0.12	0.01
13.	0.17	0.16	0.00
14.	0.20	0.19	0.00
15.	0.23	0.22	0.01
16.	0.27	0.25	0.01
17.	0.30	0.28	0.03
18.	0.34	0.31	0.03
19.	0.38	0.35	0.03
20.	0.42	0.40	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.035 FOR X = 24

SAMPLE LPG CALCULATIONS

Data Set No.	31	32	33	34	35
Ave. Req Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	.10	.50	1.00	1.50	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.25	.25	.25	.25	.25
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	.05	.25	.50	.75	1.00
Alpha	16.00	16.00	16.00	16.00	16.00
Beta	2.00	2.00	2.00	2.00	2.00
LPG Moments					
Mean	.80	4.00	8.00	12.00	16.00
CV	2.11	.97	.71	.60	.53
Skewness	3.52	1.46	.95	.73	.60
Kurtosos	21.73	6.40	4.53	3.92	3.64

Percentage Points	31		32		33		34		35	
	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	0	1	3	4	7	8	11	12	15	15
.60	0	2	4	5	8	9	13	13	17	18
.70	1	2	5	6	10	11	15	15	19	20
.80	1	2	7	7	12	13	17	19	23	23
.85	2	2	8	8	14	14	19	20	25	26
.90	3	5	1 9	9	16	16	22	22	27	28
.95	4	5	12	12	19	19	25	26	32	32
.97	5	6	13	3	21	21	28	28	35	34
.99	8	8	17	16	26	25	34	33	41	40

Table II-8. Demand Rate Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.10
AVE LEAD TIME	8.00
C OF V OF LT	0.25

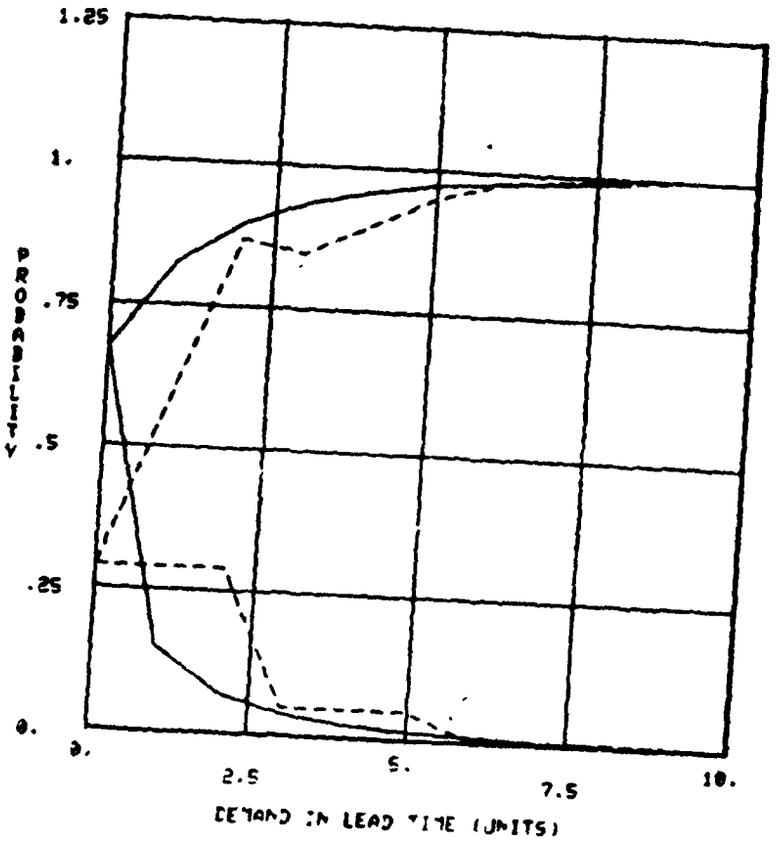
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	0.80
C OF V OF DLT	2.11
SKEWNESS	3.52
KURTOSIS	21.73

LPG PARAMETERS

THETA	0.72
LAMBDA	0.05
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.67	0.29	0.38
1.	0.82	0.58	0.24
2.	0.89	0.87	0.03
3.	0.93	0.85	0.09
4.	0.96	0.90	0.06
5.	0.97	0.95	0.02
6.	0.98	0.98	-0.00
7.	0.99	0.99	-0.00
8.	0.99	0.99	-0.00



EXACT LPG PROBABILITIES
SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.384 FOR X = 0

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	0.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

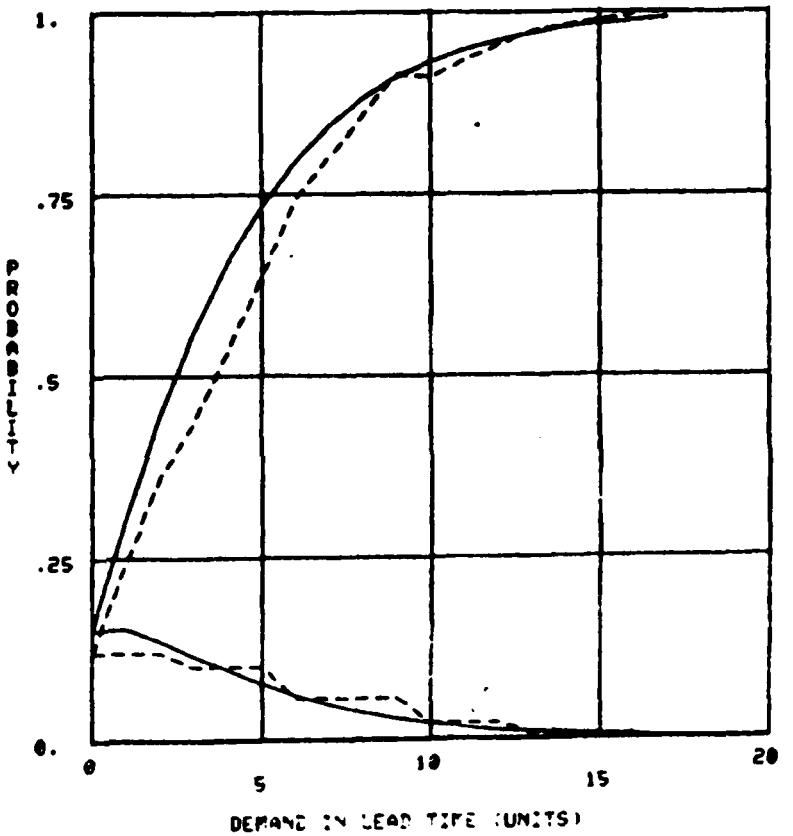
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	4.00
C OF V OF DLT	0.97
SKEWNESS	1.46
KURTOSIS	6.40

LPG PARAMETERS

THETA	0.72
LAMBDA	0.25
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.15	0.12	0.03
1.	0.31	0.24	0.06
2.	0.44	0.36	0.08
3.	0.56	0.43	0.13
4.	0.66	0.53	0.12
5.	0.73	0.64	0.10
6.	0.79	0.74	0.05
7.	0.84	0.80	0.04
8.	0.88	0.86	0.02
9.	0.91	0.91	-0.01
10.	0.93	0.91	0.02
11.	0.95	0.93	0.02
12.	0.96	0.95	0.01
13.	0.97	0.97	-0.00
14.	0.98	0.98	-0.00
15.	0.98	0.99	-0.00
16.	0.99	0.99	-0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.126 FOR X = 3

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	1.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

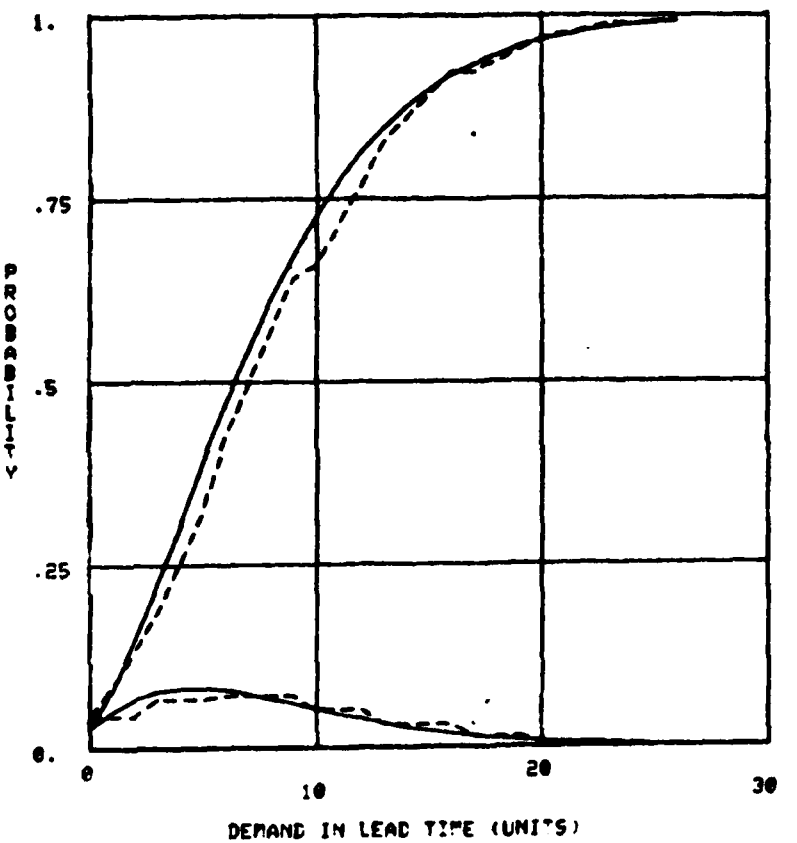
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	8.00
C OF V OF DLT	0.71
SKEWNESS	0.95
KURTOSIS	4.53

LPG PARAMETERS

THETA	0.72
LAMBDA	0.50
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.03	0.04	-0.02
1.	0.08	0.09	-0.01
2.	0.15	0.13	0.02
3.	0.23	0.19	0.04
4.	0.31	0.25	0.05
5.	0.39	0.32	0.07
6.	0.47	0.43	0.04
7.	0.54	0.50	0.05
8.	0.61	0.57	0.04
9.	0.67	0.64	0.03
10.	0.73	0.66	0.07
11.	0.77	0.71	0.06
12.	0.81	0.77	0.04
13.	0.85	0.83	0.02
14.	0.87	0.86	0.02
15.	0.90	0.89	0.01
16.	0.92	0.92	-0.00
17.	0.93	0.92	0.01
18.	0.95	0.94	0.01
19.	0.96	0.95	0.00
20.	0.97	0.97	-0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.070 FOR X = 5

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	1.50
AVE LEAD TIME	8.00
C OF V OF LT	0.25

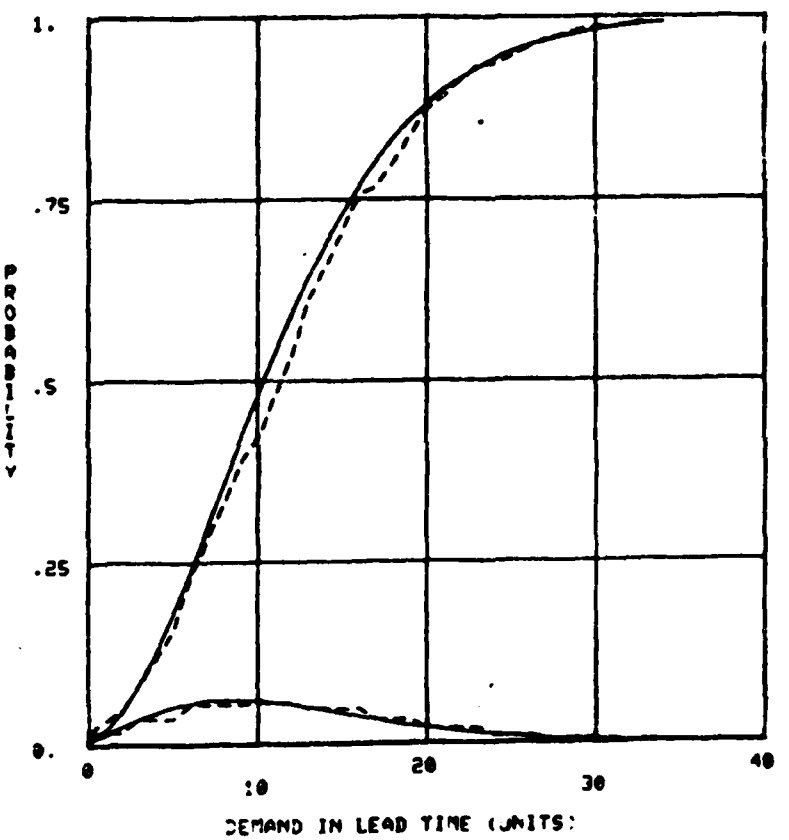
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	12.00
C OF V OF DLT	0.60
SKEWNESS	0.73
KURTOSIS	3.92

LPG PARAMETERS

THETA	0.72
LAMBDA	0.75
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.01
1.	0.02	0.03	-0.01
2.	0.05	0.05	-0.00
3.	0.08	0.08	0.00
4.	0.13	0.12	0.01
5.	0.18	0.15	0.03
6.	0.24	0.23	0.01
7.	0.30	0.28	0.02
8.	0.36	0.34	0.02
9.	0.42	0.39	0.03
10.	0.48	0.42	0.06
11.	0.54	0.48	0.06
12.	0.59	0.53	0.06
13.	0.64	0.61	0.03
14.	0.69	0.66	0.03
15.	0.73	0.71	0.02
16.	0.77	0.76	0.01
17.	0.80	0.77	0.04
18.	0.83	0.82	0.03
19.	0.86	0.83	0.02
20.	0.88	0.87	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.062 FOR X = 11

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

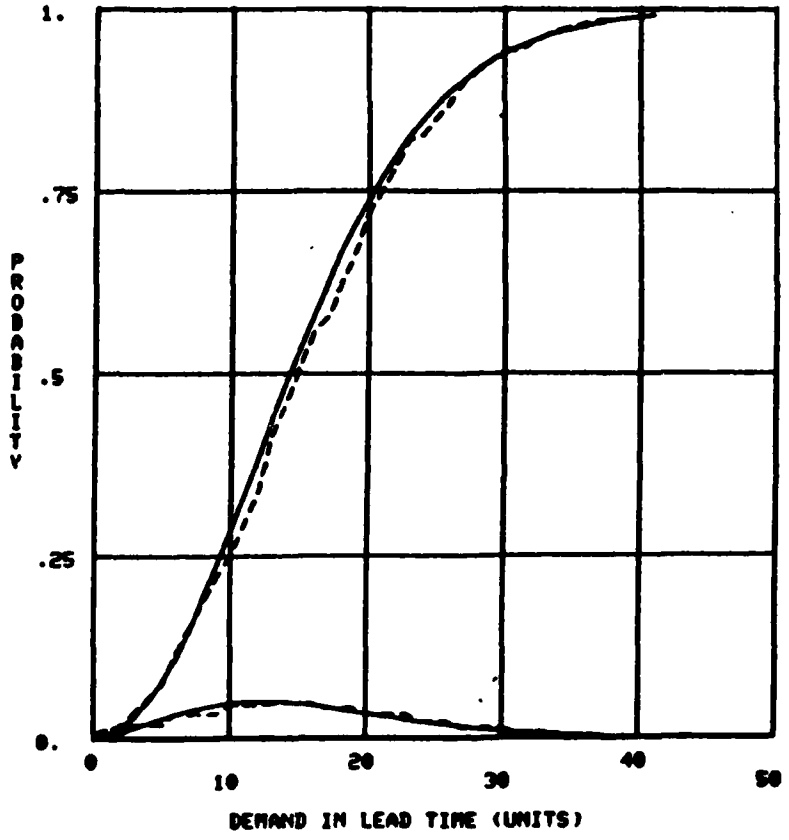
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

SAMPLE LPG CALCULATIONS

Date Set No.	36	37	38	39	40
Ave. Req. Size	2.00	2.00	2.00	2.00	2.00
Ave. Demand/Mo.	2.00	2.00	2.00	2.00	2.00
Mean Lead Time	8.00	8.00	8.00	8.00	8.00
CV of Lead Time	.01	.25	.50	.75	1.00
LPG Parameter					
Theta	.72	.72	.72	.72	.72
Lambda	1.00	1.00	1.00	1.00	1.00
Alpha	10,000	16.00	4.00	1.78	1.00
Beta	1,250	2.00	.50	.22	.13
LPG Moments					
Mean	16.00	16.00	16.00	16.00	16.00
CV	.47	.53	.69	.88	1.10
Skewness	.80	.60	.65	1.03	1.55
Kurtosis	3.96	3.64	3.69	4.86	7.10

	36		37		38		39		40	
Percentage Points	LPG	NB	LPG	NB	LPG	NB	LPG	NB	LPG	NB
.50	15	16	15	15	14	14	12	13	10	11
.60	17	18	17	18	17	18	16	16	14	15
.70	19	20	19	20	20	20	19	20		
.80	22	22	23	23	24	25	26	23	27	27
.85	24	25	25	26	27	27	30	26	32	33
.90	26	27	27	28	31	31	35	35	39	40
.95	30	29	32	32	37	37	44	44	51	51
.97	32	32	35	34	41	41	50	50	60	61
.99	38	36	41	40	50	51	64	63	80	80

Table II-9. Lead Time Variability Sensitivity for HI Base Case

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.01

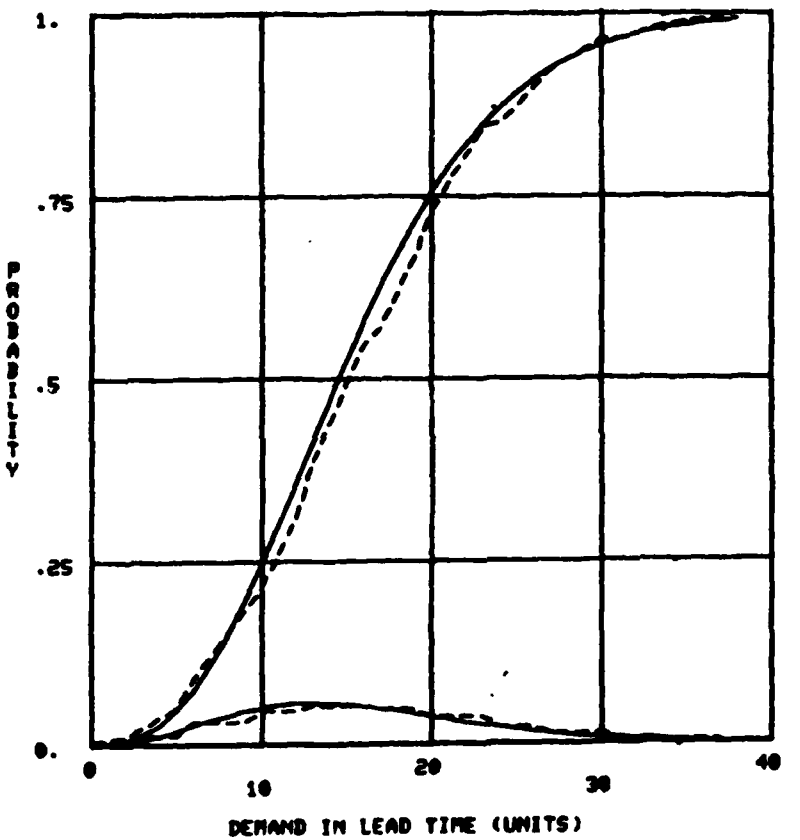
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.47
SKEWNESS	0.80
KURTOSIS	3.96

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	10000.00
BETA	1250.00

X	FX	FNBX	DIFF
0.	0.00	0.00	-0.00
1.	0.00	0.01	-0.01
2.	0.01	0.01	-0.01
3.	0.01	0.02	-0.01
4.	0.03	0.04	-0.01
5.	0.05	0.05	-0.00
6.	0.07	0.09	-0.02
7.	0.11	0.12	-0.01
8.	0.15	0.15	-0.00
9.	0.20	0.18	0.01
10.	0.25	0.21	0.03
11.	0.30	0.26	0.04
12.	0.36	0.31	0.05
13.	0.42	0.39	0.03
14.	0.47	0.44	0.03
15.	0.53	0.49	0.03
16.	0.58	0.55	0.03
17.	0.63	0.57	0.06
18.	0.67	0.62	0.05
19.	0.72	0.67	0.05
20.	0.76	0.73	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.058 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.25

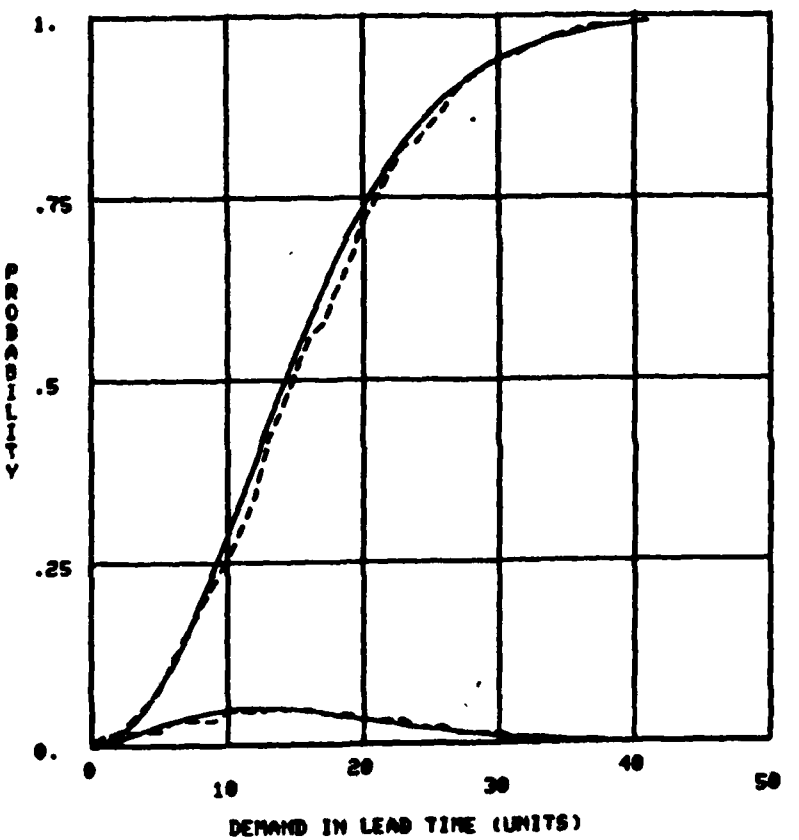
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.53
SKEWNESS	0.60
KURTOSIS	3.64

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	16.00
BETA	2.00

X	FX	FNBX	DIFF
0.	0.00	0.01	-0.01
1.	0.01	0.01	-0.01
2.	0.02	0.02	-0.00
3.	0.03	0.04	-0.01
4.	0.05	0.05	-0.00
5.	0.08	0.07	0.00
6.	0.11	0.12	-0.01
7.	0.15	0.15	-0.00
8.	0.19	0.19	0.00
9.	0.24	0.22	0.02
10.	0.29	0.25	0.04
11.	0.34	0.30	0.04
12.	0.39	0.34	0.05
13.	0.44	0.41	0.02
14.	0.49	0.46	0.03
15.	0.54	0.51	0.03
16.	0.58	0.56	0.02
17.	0.62	0.58	0.05
18.	0.67	0.62	0.04
19.	0.70	0.66	0.04
20.	0.74	0.72	0.02



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.047 FOR X = 17

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.50

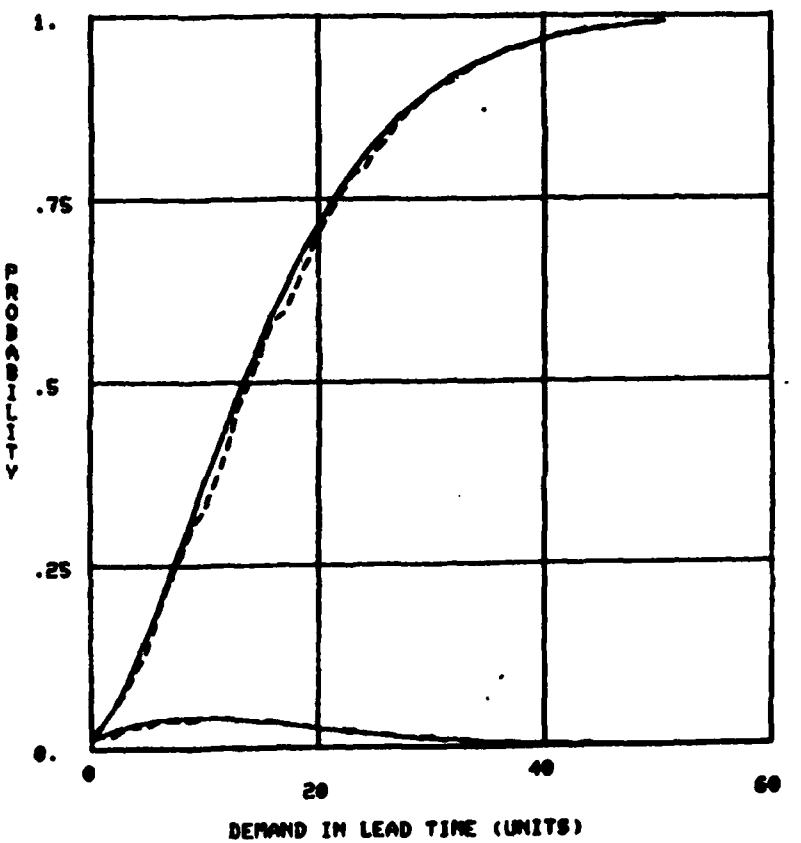
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.69
SKEWNESS	0.65
KURTOSIS	3.69

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	4.00
BETA	0.50

X	FX	FNBX	DIFF
0.	0.01	0.02	-0.00
1.	0.03	0.03	-0.00
2.	0.06	0.05	0.00
3.	0.09	0.08	0.01
4.	0.12	0.11	0.01
5.	0.16	0.13	0.02
6.	0.19	0.19	0.01
7.	0.24	0.23	0.01
8.	0.28	0.27	0.01
9.	0.32	0.30	0.02
10.	0.36	0.33	0.04
11.	0.40	0.37	0.04
12.	0.45	0.41	0.04
13.	0.48	0.47	0.02
14.	0.52	0.51	0.02
15.	0.56	0.54	0.02
16.	0.59	0.58	0.01
17.	0.63	0.60	0.03
18.	0.66	0.63	0.03
19.	0.69	0.66	0.03
20.	0.72	0.70	0.01



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.037 FOR X = 11

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	0.75

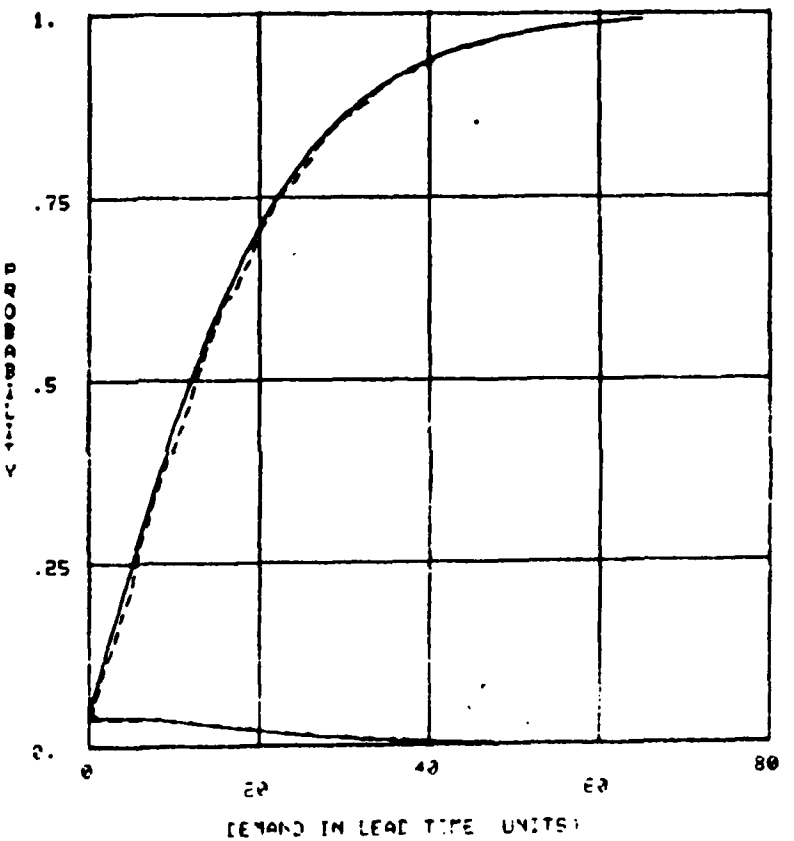
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	0.88
SKEWNESS	1.03
KURTOSIS	4.87

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	1.78
BETA	0.22

X	EX	ENB	DIFF
0.	0.05	0.04	0.01
1.	0.09	0.08	0.01
2.	0.13	0.11	0.01
3.	0.17	0.14	0.03
4.	0.21	0.18	0.03
5.	0.25	0.22	0.03
6.	0.29	0.28	0.01
7.	0.33	0.31	0.01
8.	0.37	0.35	0.01
9.	0.40	0.39	0.01
10.	0.44	0.41	0.03
11.	0.47	0.44	0.03
12.	0.50	0.47	0.03
13.	0.53	0.52	0.01
14.	0.56	0.55	0.01
15.	0.59	0.58	0.01
16.	0.62	0.61	0.01
17.	0.64	0.62	0.02
18.	0.67	0.65	0.02
19.	0.69	0.67	0.02
20.	0.71	0.70	0.01



-------- EXACT LPG PROBABILITIES
- - - - - SCALED NEGATIVE BINOMIAL

MAX DIFFERENCE= 0.031 FOR X = 10

Best available

DEMAND IN LEAD TIME PARAMETERS

AVE REQ SIZE	2.00
AVE MONTHLY DEMAND	2.00
AVE LEAD TIME	8.00
C OF V OF LT	1.00

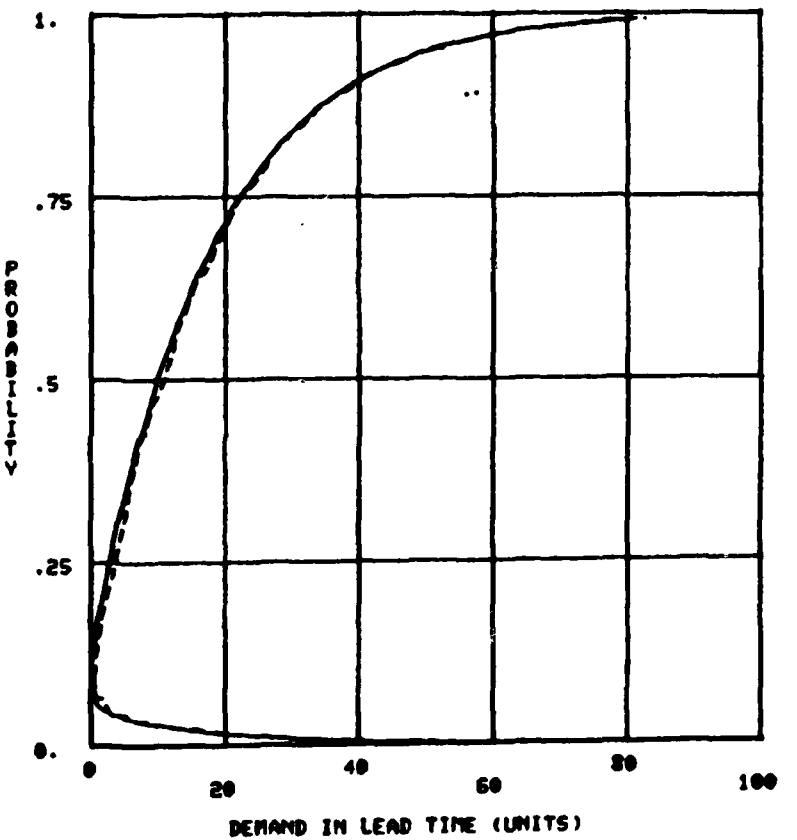
LPG DISTRIBUTION PARAMETERS

MEAN DEMAND	16.00
C OF V OF DLT	1.10
SKEWNESS	1.55
KURTOSIS	7.10

LPG PARAMETERS

THETA	0.72
LAMBDA	1.00
ALPHA	1.00
BETA	0.13

X	FX	FNBX	DIFF
0.	0.11	0.07	0.05
1.	0.17	0.13	0.04
2.	0.22	0.20	0.02
3.	0.26	0.22	0.04
4.	0.30	0.26	0.04
5.	0.34	0.31	0.03
6.	0.38	0.36	0.01
7.	0.41	0.40	0.01
8.	0.44	0.43	0.01
9.	0.47	0.47	0.01
10.	0.50	0.48	0.03
11.	0.53	0.51	0.02
12.	0.56	0.53	0.02
13.	0.58	0.57	0.01
14.	0.60	0.59	0.01
15.	0.62	0.62	0.01
16.	0.64	0.64	0.00
17.	0.66	0.65	0.02
18.	0.68	0.67	0.02
19.	0.70	0.69	0.01
20.	0.72	0.71	0.00



-------- EXACT LPG PROBABILITIES
- - - - SCALED NEGATIVE BINOMIAL

MAX CDF DIFFERENCE= 0.046 FOR X = 0

Appendix B

LPG Program Source Listings

```
 LIST

10 REM LPG.S--COMPUTE EXACT LPG PROB USING 6/19/81 RECURSION
20 REM
30  DIM T(200),T2(200)
40 REM
50 REM    T(K) = T(X,K) FOR THE CURRENT X
60 REM   T2(K) = T(X-1,K) FROM PREVIOUS X CALCULATION
70     FILES LPG.B
80     SCRATCH #1
90 REM          INPUT PARAMETERS
91     PRINT "LPG.S----   DEBUG ?  PRINT F(X)? (1 OR 0)"
93      INPUT B8, B7
94  REM
100         GOSUB 180
110 REM          DO LPG RECURSION
120     GOSUB840
130REM           DO SCALLED BINOMIAL CALCULATIONS
140     GOSUB 3180
150     PRINT "CONTINUE? (Y OR N)"
155     INPUT A$
160     IF A$="Y" THEN 90
170     STOP
180 REM
181     PRINT
182    PRINT
190 PRINT "LPGO.S--EXACT LPG PROB CALCULATIONS USING RECURSION"
191     PRINT
192     PRINT
200  REM
210  PRINT "INPUT AVE REQ, E(D), E(LT), C.OF.V OF LT "
220  INPUT R0,B1,E1,C
230     WRITE #1,R0,B1,E1,C
240 REM
250 REM               SOLVE FOR THETA = T1
260   GOSUB 1780
270   T1=0
280 REM
290 REM           ESTIMATE PARAMETERS FOR LPG
300 REM         S = STD DEV,  B = VAR TO MEAN RATIO
310    S = C*E1
320    B = S*S/E1
330     U1= D1*E1
340 REM
350   B1=1/B
360   A1 =B1*E1
370   R1=-T1/( (1-T1) *LOG(1-T1) )
380   L1 = U1/(E1*R1)
390 REM
400   PRINT "AVE REQ SIZE =",R1
410 PRINT
420    PRINT "THETA =",T1,"LAMBDA =",L1
430    PRINT "ALPHA=",   A1,"BETA =",B1
440 REM
```

55

```
450 REM
460     M1=-T1/((1-T1)*LOG(1-T1)*L1*A1/B1)
470 PRINT
480 PRINT "MEAN =",M1
490 PRINT
500REM      ESTIMATE THE FIRST FOUR MOMENTS M1,M2,M3,M4 OF THELPG DISTRIBUTION
510      B1 = B1*(1-T1)
520     C4 = -L1*T1/(LOG(1.-T1))
530 REM
540REM
550      M1 = A1*C4/D1
560REM
570      M2 = A1*C4*(B1+C4)/D1^2
580REM
590      M3 = A1*C4*((B1^2)*(1+T1) +2*C4^2)/B1^3
600REM
610     M4 = B1^3*(1+4*T1+T1^2) + B1^2*C4*(3.*A1 + 1.)
620      M4 = M4 + 6*B1*C4*C4*A1 +C4^3*(3*A1+6.)
630      M4 = A1*C4*M4/D1^4
640REM
650      PRINT "MOMENTS"
660      PRINT "M1      M2      M3      M4"
670      PRINT M1,M2,M3,M4
680REM
690REM          COMPUTE STANDARDIZED MOMENTS
700      PRINT
710      S = SQR(M2)
720      PRINT "COEF OF VAR =", S/M1
730      PRINT "M3/S^3= ", M3/S^3
740      PRINT "M4/S^4= ", M4/S^4
750REM
760REM           OUTPUT LPG PARAMETERS TO FILE
770 REM
780           WRITE #1,M1,S/M1,M3/S^3,M4/S^4
790           WRITE #1,T1,L1,A1,B1
800           RETURN
810REM ***************************************************************
820REM              LPG RECURSION CALCULATION
830 REM***************************************************************
840 REM
850 REM           REDEFINE C FOR PROBABILITY CALCULATIONS
860 REM
870      C= -L1/LOG(1.-T1)
880      C1 = C/(L1 + B1)
890 REM
900 REM           SET LIMIT ON X = 200
910 REM
920     L2=200
930 REM
940     C2=0
950 REM
960 REM           SET CONSTANTS FOR USE IN RECURSION
```

```
970 REM
980     S7=1
990    H1=(B1/(L1+B1))^A1
1000  H1=S7*H1
1010 REM            S7 = SCALE FACTOR
1020    H2=(C/(L1+B1))
1030 REM
1040  PRINT"C,C1,H1,H2",C;C1;H1;H2
1050 REM
1060 REM          EVALUATE H(X=0)
1070 REM
1080   T(0) = 0
1090    T2(0) = H1
1100    S1 = H1
1110     C2 = C2 + S1/S7
1120 REM
1130   PRINT
1131     WRITE#1,0,S1,C2
1140     IF D7<= 0 THEN 1200
1141     PRINT
1150     PRINT "          X            H(X)          F(X)"
1151     PRINT
1160    PRINT 0;S1;C2
1170  REM
1180 REM
1190 REM      EVALUATE H(X) FOR X > 0
1200 REM
1210    FOR X = 1 TO L2
1220    S1 = 0
1230     T(0)=0
1240 REM
1250     T(X)=(T1*H2*(A1+X-1)/X)*T2(X-1)
1260 REM
1270    IF X < 2 GOTO 1380
1280 REM
1290 REM
1300 REM
1310    FOR K=1 TO X-1
1320     T(K)=(T1/X) * (H2*(A1+K-1)*T2(K-1) +(X-1)*T2(K) )
1330     S1=S1+T(K)
1340         IF D8 <= 0 THEN 1360
1350     PRINT"X,K,T(X,K),S1=>",X;K;T(K);S1
1360    NEXT K
1370 REM          PICK UP T(X,X) TERM IN SUM
1380     S1=S1 + T(X)
1390 REM
1400 REM          PRINT TOTALS FOR H(X)
1410 REM
```

```
1420      C2 = C2 + S1/S7
1430     PRINT X;S1;C2
1440        WRITE #1,X,S1,C2
1450 REM
1460 REM          IF CUM PROB EXCEEDS .99, STOP
1470 REM
1480      IF C2 > .99 GOTO 1670
1490 REM
1500 REM          PRINT T(K) TERMS FOR DEBUGGING
1510 REM
1520     IF D8 <= 0 THEN 1560
1530   FOR K=0 TO X
1540     PRINT "X,K,T(X,K) =>",X;K;T(K)
1550    NEXT K
1560 REM
1570 REM
1580 REM          RECORD T(K) VALUES FOR USE IN NEXT PASS
1590 REM
1600     FOR K=0 TO X
1610       T2(K)=T(K)
1620      NEXT K
1630 REM
1640 REM----------------------END OF X LOOP
1650      NEXT X
1660 REM
1670     WRITE #1,-99,-99,-99 "    END OF LPG"
1680     RETURN
1690 REM*****************************************END OF LPG RECURSION
1700 REM
1710 REM
1720 REM
1730 REM---------------------------------AVE REQ SIZE EQUATION
1740 DEF FNR(Q) = -Q/( (1-Q)*LOG(1-Q)  )
1750 REM
1760 REM-----------------------------------
1770 REM              BINARY SEARCH ROUTINE
1780 REM -----------------------------------
1790 REM
1800 REM     SOLVE FOR THE VALUE OF Q WHICH GIVES AND
1810 REM       AVE. REQUISITION SIZE OF RO
1820 REM
1830 REM    SET UP END POINTS FOR SEARCH
1840 REM
1850    K= 0
1860   Q9=.999
1870   R9=FNR(Q9)
1880   Q1=.001
1890   R1=FNR(Q1)
1900     GOTO 2080
1910 REM
```

```
1920 REM     CHECK IF R0 <= R
1930 REM
1940      IF R0 > R THEN 2000
1950  REM
1960  REM          RE-SET TOP OF INTERVAL
1970     R9=R
1980     Q9=Q
1990      GOTO 2080
2000 REM
2010 REM          RE-SET BOTTON OF INTERVAL
2020     R1=R
2030      Q1=Q
2040       GOTO 2080
2050 REM
2060 REM          HALVE THE INTERVAL, AND RE-EVALUATE THE FUNCTION
2070 REM
2080      K=K+1
2090     Q=(Q1+Q9)/2
2100      R=FNR(Q)
2120 REM
2130 REM          IF R IS WITHIN .001 OF R0, THEN RETURN
2140 REM
2150      IF ABS(R-R0) < .001 THEN 2180
2160      IF K > 15 THEN 2180
2170       GOTO 1940
2180    RETURN
2190  REM
2200REM          LPG.S---LP PROBABILITY CALCULATIONS
2210REM
2220REM        USELPG.D AS THE OUTPUT FILE
2230REM
2240   REM
2250    SCRATCH #1
2260     PRINT "THETA, LAMBDA, ALPHA, BETA, DEBUG?"
2270     INPUT T1,L1,A1,B1,D8
2280     PRINT "EXACT LPG PROB"
2290    GOSUB 2420
2300     PRINT "SCALED MES. BIN"
2310     GOSUB 3230
2320     PRINT
2330     PRINT "CONTINUE?(Y OR N)"
2340     INPUT A$
2350     IF A$="Y" GOTO 2260
2360     STOP
```

```
2370 REM LPG.S--MASTER LPG ROUTINE LIBRARY
2380 REM LPG1.S
2390 PRINT "THIS PROGRAM COMPUTES EXACT PROB FOR THE LPG DIST"
2400 DIM Y(100,100)
2410 INPUT T1,L1,A1,B1
2420 PRINT "THETA =";T1,"LAMBDA =";L1"ALPHA =";A1,"BETA =";B1
2430      WRITE #1,1,2,3,4
2440      WRITE #1,T1,L1,A1,B1
2450 C = - L1 /LOG(1 -T1)
2460 M1 = -T1/((1-T1)*LOG(1-T1))*L1*A1/B1
2470 C1 =C/(L1 + B1)
2480 L2 = INT(100*M1)
2490 C2 =0
2500 FOR X =0 TO L2
2510 Y(X,0)=0
2520 F1 =1
2530 IF X<2 THEN 2570
2540 FOR K = 1 TO X-1
2550 F1 = F1*K
2560 NEXT K
2570 Y (X,X) = 1./F1
2580 F1 = 1
2590 IF X<> 0 THEN 2630
2600 S1 =(B1/(L1 +B1))^A1
2610 GO TO 2740
2620 REM
2630 S1 =0
2640 FOR K = 1 TO X
2650 IF X = 1 THEN 2670
2660 Y(X,K)=Y(X-1,K-1)/(X-1) + Y(X-1,K)
2670 F1 = 1
2680 FOR J = 0 TO K-1
2690 F1 = F1*(A1 + J)*C1
2700 NEXT J
2710 S1 = S1 + Y(X,K)*F1
2720 NEXT K
2730 S1 = S1*(B1/(L1 + B1))^A1*T1^X/X
2740 C2 = C2 + S1
2750 IF D8 <=0 THEN 2780
2760 PRINT X,S1,C2
2770 WRITE #1,X,S1,C2
2780 IF C2 > .99  THEN 2800
2790 NEXT X
2800 WRITE #1,-99,-99,-99
2810 RETURN
```

```
2820 REM
2830 REM ************************************************************
2840 REM LPG2.S
2850 PRINT "SCALED POISSON"
2860 INPUT T1,L1,A1,B1
2870 PRINT "THETA =";T1,"LAMBDA =";L1,"ALPHA =";A1,"BETA =";B1
2880 C = -L1/LOG(1-T1)
2890 C3= 1./(1.-T1)
2900 K1 = T1*C
2910 C4 = (B1/(K1+B1))^A1
2920 U = K1/(K1+B1)
2930 C2 = 0
2940 FOR M = 0 TO 1000
2950 P1 = 1
2960 IF M<>0 THEN 3000
2970     P1 = C4
2980     GOTO 3040
2990 REM
3000 FOR I = 0 TO M-1
3010 P1 = P1*((A1+M-I-1)/(M-I)*U)
3020 NEXT I
3030 P1 = P1*C4
3040 C2 = C2 + P1
3050   IF B8<=0 THEN 3080
3060 PRINT M;C3*M;P1;C2
3070 WRITE #1,C3*M,P1,C2
3080  IF C2>.99  THEN 3100
3090 NEXT M
3100 RETURN
3110 RETURN
3120 REM ************************************************************
3130 REM
3140 REM LPG3.S
3150 PRINT "THIS PROGRAM COMPUTES APP PROB FOR LPG DIST USING SCALED BIN"
3160 INPUT T1,L1,A1,B1
3161     PRINT
3170 PRINT "THETA =";T1,"LAMBDA =";L1,"ALPHA =";A1,"BETA =";B1
3180     PRINT
3190     PRINT "NEG BINOMIAL PROBABILITIES"
3200     PRINT
3210    PRINT "          X          P          F"
3220     PRINT
3230 C = 1/(1.-T1)
3240 K1 = -T1*L1/LOG(1-T1)
3250 C4 = (B1/(K1+B1))^A1
3260 U = K1/(K1+B1)
3270 C2 = 0
3280 FOR M = 0 TO 1000
3290 P1 = 1
3300 IF M<>0 THEN 3370
3310 P1 = C4
3320 A = -1
3330  B = INT(C/2 +0.5)
3340 S2 = P1/(C/2+1)
3350 GOTO 3440
```

```
3360 REM
3370 FOR I = 0 TO N-1
3380 P1 = P1*((A1+N-I-1)/(N-I)*W)
3390 NEXT I
3400 P1 = P1*C4
3410 A = INT((2*N-1)*C/2+0.5)
3420 B = INT((2*N+1)*C/2+0.5)
3430 S2 = P1/C
3440 L3 = C2
3450 FOR X = A+1 TO B
3460 F3 = C2 + S2*(X-A)
3470 F2 = F3 - L3
3480 L3 = F3
3490     IF B7 <= 0 THEN 3520
3500 PRINT X,F2,F3
3510 WRITE#1,X,F2,F3
3520 NEXT X
3530 C2 = C2 + P1
3540 IF C2 > .99  THEN 3560
3550 NEXT N
3560   WRITE#1,-99,-99,-99,"END OF NB"
3570 RETURN
3580 REM
3590 REM ************************************
3600 REM LPG4.S
3610 REM          COMPUTE REQUISITION SIZE R VS THETA TABLE
3620 REM
3630 FOR T1 = .01 TO .99 STEP .01
3640    B = -(1-T1)*LOG(1-T1)
3650     R = T1/B
3660    PRINT T1,R
3670 NEXT T1
3680 STOP
3690 REM ************************************************************
3700REM DEFINITIONS IN HAHNIAS' PROGRAM LPG1.S
3710 REM                  DEFINITIONS IN HAHNIAS' PROGRAM LPG1.S
3720 REM
3730 REM          LAMB          L1
3740 REM          ALPH          A1
3750 REM          BET           B1
3760 REM          MEAN          M1
3770 REM          CNST          C1
3780 REM          LIN           L2
3790 REM          CUM           C2
3800 REM          FACT          F1
3810 REM          SUM           S1
3820 REM          KN            K1
3830 REM          PROD          P1
3840 REM          CN            C3
3850 REM          CONST         C4
3860 REM          LAST          L3
3870 REM          FX            F2
3880 REM          FFX           F3
3890 REM          SL           S2
3900 END
```

```
IDEN LPGSKU.P
FILE 11
DASH 1
LSYM 1
GRID 1
TLAB MEAN  VS DATA SET NO.
BLAB FOR SAMPLE LPG PARAMETER SETS
YLAB MEAN  .
XLAB DATA SET NO.
XCOL 2,2,49  2,2,49  2,2,49  2,2,49
YCOL 3,2,49
PLOT
PLOT
TLAB STD. DEV. VS DATA SET NO.
YLAB STD. DEV
YCOL 6,2,49
PLOT
TLAB SKEWNESS**2 VS KURTOSIS
XLAB KURTOSIS
YLAB SKEWNESS**2
XCOL 5,2,7  5,8,13  5,14,19  5,20,25  5,26,31
YCOL 4,2,7  4,8,13  4,14,19  4,20,25  4,26,31
SCALE  0,10      0,5
PLOT
CONT 1
XCOL  5,32,37   5,38,43  5,44,49
YCOL  4,32,37   4,38,43  4,44,49
PLOT
CONT 0
BLAB LPG VALUES FOR E(D)=0.5
XCOL 5,2,7  5,8,13  5,14,19  5,20,25
YCOL 4,2,7  4,8,13  4,14,19  4,20,25
PLOT
BLAB LPG VALUES FOR E(D)=2.0
XCOL 5,26,31  5,32,37  5,38,43  5,44,49
YCOL 4,26,31  4,32,37  4,38,43  4,44,49
PLOT
STOP
```

OLD 11

1	2 (Thick Vol)	3 MEAN	4 Skew^2	5 Kurto	6 Std Dev	7	8 Skew
10	2	3.000	25.000	6.000	12.000	10	5.000
20	1	4.000	0.168	3.230	2.240	10	0.410
30	1	4.000	0.168	3.230	2.240	10	0.410
40	2	4.000	2.132	6.400	3.880	10	1.460
50	3	4.000	8.762	16.450	6.520	10	2.960
60	4	4.000	25.806	48.130	10.520	10	5.080
70	5	4.000	66.912	103.790	16.600	10	8.180
80	6	2.000	4.666	10.220	2.700	10	2.160
90	6	2.000	4.666	10.220	2.700	10	2.160
100	7	3.000	2.958	7.670	3.330	10	1.720
110	8	4.000	2.132	6.400	3.880	10	1.460
120	9	5.000	1.638	5.650	4.350	10	1.280
130	10	6.000	1.300	5.150	4.860	10	1.140
140	11	0.800	12.390	21.730	1.688	10	3.520
150	11	0.800	12.390	21.730	1.688	10	3.520
160	12	4.000	2.132	6.400	3.880	10	1.460
170	13	8.000	0.903	4.530	5.680	10	0.950
180	14	12.000	0.533	3.920	7.200	10	0.730
190	15	16.000	0.360	3.640	8.480	10	0.600
200	16	4.000	2.592	6.840	3.760	10	1.610
210	16	4.000	2.592	6.840	3.760	10	1.610
220	17	4.000	2.132	6.400	3.880	10	1.460
230	18	4.000	1.464	5.550	4.240	10	1.210
240	19	4.000	1.277	5.150	4.800	10	1.130
250	20	4.000	1.664	5.760	5.480	10	1.290
260	21	16.000	0.073	3.130	5.760	10	0.270
270	21	16.000	0.073	3.130	5.760	10	0.270
280	22	16.000	0.360	3.640	8.480	10	0.600
290	23	16.000	1.822	5.970	13.440	10	1.350
300	24	16.000	5.954	12.320	21.280	10	2.440
310	25	16.000	16.160	27.700	33.440	10	4.020
320	26	8.000	0.903	4.530	5.680	10	0.950
330	26	8.000	0.903	4.530	5.680	10	0.950
340	27	12.000	0.533	3.920	7.200	10	0.730
350	28	16.000	0.360	3.640	8.480	10	0.600
360	29	20.000	0.270	3.480	9.800	10	0.520
370	30	24.000	0.221	3.380	11.040	10	0.470
380	31	0.800	12.390	21.730	1.688	10	3.520
390	31	0.800	12.390	21.730	1.688	10	3.520
400	32	4.000	2.132	6.400	3.880	10	1.460
410	33	8.000	0.903	4.530	5.680	10	0.950
420	34	12.000	0.533	3.920	7.200	10	0.730
430	35	16.000	0.360	3.640	8.480	10	0.600
440	36	16.000	0.640	3.960	7.520	10	0.800
450	36	16.000	0.640	3.960	7.520	10	0.800
460	37	16.000	0.360	3.640	8.480	10	0.600
470	38	16.000	0.423	3.690	11.040	10	0.650
480	39	16.000	1.061	4.860	14.080	10	1.030
490	40	16.000	2.403	7.100	17.600	10	1.550

Plot Program and Input Data For LPG Skewness vs Kurtosis Plots.